PARTICULAR ASPECTS CONCERNING INSTABILITY AT INTERFACE LCAK STEEL - SLAG (CaO-Al₂O₃) DUE TO TRANSFER OF SULPHUR, EVALUATED USING THE MODEL REICHENBACH-LINDE ADAPTED

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ABSTRACT

Based on the model of Reichembach and Linde, adapted to the interface in the system LCAK steel-(CaO-Al₂O₃) slag at temperature 1873.15K, it is shown that transfer of sulphur from steel to slag could promote interfacial Marangoni instability, proven by positive and relative small values of Marangoni dimensionless number, related to slag. Positive values of critical Marangoni number have been found in the rage of dimensionless wavenumber \( a < a_c \), \( a_0 = 2.881255 \pm 5 \times 10^{-6} \) and \( a_\infty = 3.0058 - 3.006 \) (asymptote) for a characteristic length \( d = 100 \mu m \). In the range of values of the dimensionless wavenumber \( a = 2.9 - 3.0 \), corresponding to wavelength \( \lambda = 216.66 - 209.44 \mu m \), a relation \( Ma_{c, slag} = (0.0957a + 0.618 + \frac{1}{L_S \cdot D^*}) \cdot e^{16.89a^2 - 1947.51a + 1331.7} \), containing also the sulphur partition ratio between slag and steel \( L_S \) and ratio of diffusion coefficients of sulphur in slag and steel \( D^* \), was established giving values of critical Marangoni dimensionless number. Values \( Ma_{c, slag} = 20 - 5200 \) are obtained for \( L_S = 10 - 1000 \), supporting the idea that slag is active in the dynamics of interface steel-slag with consequences on the steel quality and kinetics of important processes of refining, contrary to certain evaluations seeming to be insufficiently substantiated. Stationary instability in the form of short wavelength waves could develop in the mentioned conditions or, considering the asymptote found at \( a_\infty = 3.0058 - 3.006 \) oscillatory waves could develop.

KEYWORDS: interface, steel, slag, sulphur transfer

1. Introduction

Capillarity during steelmaking and refining is recognized as an important phenomenon in hydrodynamics of interface in liquid systems in general, including those formed by ferrous alloys, slags and atmospheres[1]-[7], but the analysis of its manifestations, according to the well known achievements in related systems, also following a philosophy close to them, is not yet approached practically. In most of cases, facts derived from technological practice and results of experiments are linked only qualitatively or empirically with certain parameters of capillarity. A more consistent analysis is imposed by needs of high quality steels characterized, among other requirements, by low and extremely low contents of harmful elements as oxygen, sulphur and phosphorus and a high cleanliness state, concerning different non-metallic inclusions.

Some qualitative considerations have been reported [2]-[5], also some quantitative but restricted to particular cases [6][7] and in both situations without treating in particular manner specific capillarity aspects related to each of fluids forming the interface.

Based on the simplified ”one layer” model applied to slag, it was shown[8][9] that sudden local variations of the sulphur content, taken as inhomogeneities, of order \( 10^{-3} + 10^{-2} \) mass% in CaO-Al₂O₃ slags, typical for desulphurization process, could excite a Marangoni instability and could lead to enough high values of solutal Marangoni number \( (Ma) \), overcoming also enough its critical threshold value \( (Ma_c \cong 80) \) established according to Pearson...
These suggest that Marangoni flow and convection could take place at a scale of several hundreds of micrometers at interface steel-slag by advection and further by convection, considering the continuity law. The respective analysis was performed in certain established conditions, ensuring the most favorable conditions, i.e. minimal conditions required for solutal Marangoni instability, based mainly on an argued scaling of the characteristic lengths and characteristic time durations of different competing physico-chemical actions at interface on slag side.

Sternling and Scriven [11], based on their linear analyze of stability and in absence of any scaling of characteristic time and length, have established seven cases when interfacial turbulence is usually promoted, in a system of two superposed immiscible fluids of semifinite heights. From them, the main situations are: 1-solute transfer out of the phase of higher viscosity; 2- solute transfer out of the phase in which its diffusivity is lower; 3- large differences in kinematic viscosity and solute diffusivity between the two phases; 4- steep concentration gradients near the interface; 5- interfacial tension highly sensitive to solute concentration. Taking into account data from the Table 1, according to statements of Sternling and Scriven statements [11], there is no interfacial convection due to sulphur transfer from steel into slag, because it is against of points 1 and 2 from above. In the same time, according to point 3 and possible to point 4, interfacial turbulence could be promoted at interface steel-slag.

Several factors and parameters oppose to instability in a system of two immiscible fluids with interface. Firstly, the presence of the heavier fluid (steel) bellow the lighter one (slag) acts to stabilize the interface. Secondly, the presence of the interfacial tension between two immiscible fluids is a stabilizing factor of interface and in a certain configuration of the system it introduces a particular minimal wavelength necessary to promote instability.

Regarding the system steel-slag, evaluations using related criteria based on a generally accepted procedure, are totally missing. In the case of desulphurization process of LCAK steels treated under CaO-Al$_2$O$_3$ slags, the missing could be due perhaps to the transfer of sulphur from steel into slag, despite this solute is a surface tension active both in slag as in steel, according to the first two criteria [11] cannot promote interfacial turbulence, contrary to the physical evidence of an observed strong turbulence accompanied by emulsification [1]. Another reason could be that in steel refining frequently intended external actions during technological process could be totally prevalent at macro-scale, shadowing the surface driven phenomena at micro-scale, further making them to seem as negligible.

When the two layer model is considered in the system liquid steel-slag, important problems appear because it is necessary to establish how the interface could be considered, sharp or diffuse. Because steel and slag are immiscible liquids of extremely different nature and properties, the interface is considered in this paper as a sharp one. This seems to be quite acceptable for stationary cases corresponding to resting state or to zero relative velocity of the liquids forming the interface, on certain substantial depths of the layers in both liquids. This option is in connection with the present analysis which is dedicated to establish the minimal conditions of the marginal hydrodynamic stability (Marangoni) when the interfacial tension is perturbed by variations of solutal origin, enough strong, continual and persistent, i.e. local fluctuation of concentration of a surface tension active solute, able to promote Marangoni flow.

The approached problems in this paper are:

i) to evaluate the numerical evolution of $Ma$, upon characteristics of wave function when sulphur is transferred from steel to slag through the interface;

ii) to evaluate the domain of positive values of $Ma$, related to slag properties, proving that possible Marangoni instability takes place at an interface steel-slag when sulphur is transferred from steel to slag.

This is done in the case when the interface between steel and slag results from boundary layers of thickness resulting from theory of momentum dissipation and the whole system is reduced by truncation at layers having such established thicknesses.

2. Adaptation of the model Reichenbach-Linde to the particular case of LCAK steel – CaO - Al$_2$O$_3$ slag interface

The main chemical reaction describing the desulphurization process of LCAK steel under CaO-Al$_2$O$_3$ slags is the following:

$$[S]^+(CaO)+2/3[Al] \leftrightarrow (CaS)+1/3(Al_2O_3) \Delta G_f^0<0 \ (I)$$

In the analysis performed in this paper the thermochemical effects at interface in both liquids will be neglected because corresponding thermal diffusion coefficients in steel and slag are higher with one to two order of magnitude than the corresponding mass diffusion coefficients, also because of the considered smallness of the concentration inhomogeneity of sulphur [7]. The analysis is focused on the capillary consequences of the main process consisting in the transfer of sulphur from steel into slag, mainly to promote the Marangoni flow and convection in slag, which at the first sight is in contradiction to behavior predicted by Sternling and Scriven model[11]
Microconvection is a flow which originates from random disturbances in the temperature or concentration field. For these random disturbances to amplify, the system needs to be unstable.

The Marangoni number for the solutocapillarity case is frequently expressed as it is shown in the rel. (1), according to [10]:

\[ Ma = \frac{\left( \frac{\partial \sigma}{\partial c} \right)_0 \cdot L}{\mu \cdot D} \]  

(1)

The form given in rel.(2) was used in the model Reichembach-Linde [12]:

\[ Ma = \frac{\left( \frac{\partial \sigma}{\partial c} \right) \cdot \frac{\partial c}{\partial z} \cdot d^2}{\mu \cdot D} \]  

(2)

Significance of factors in the rel. (1) and (2) is the following:

\( \left( \frac{\partial \sigma}{\partial c} \right)_0 \) – is the dependence of surface/interface tension on concentration; it is called frequently surface/interface concentration coefficient; it is taken as a constant value in order to be in agreement with the Boussinesq approximation extended to surface/interface tension dependence on concentration of surface active solute, procedure being necessary to perform a linear analyses;

\( c_0 \) – is a characteristic concentration or a characteristic difference of concentration \( \Delta c \) (rel.1);

\( \frac{\partial c}{\partial z} \) – is the concentration gradient (rel.2);

\( L \) – is a characteristic length (rel.1) [10];

\( d \) – unit length (rel.2) [12];

\( D \) – is the diffusivity of the surface active solute in the phase of interest;

\( \mu \) – is the dynamic viscosity of the considered phase.

Marangoni number expresses a ratio of a characteristic diffusion or diffusion-viscosity time and a characteristic time for Marangoni driven flow. Different relations of Marangoni number appear in the literature, which makes any comparison to be difficult in certain conditions. Marangoni numbers express the ratio of surface tension forces and viscous drag. In this respect, the critical value of The Marangoni dimensionless number is \( Ma \), and shows how many times the perturbation of the surface tension has to be greater than the visco-diffusion forces, in order to promote flow.

A characteristic concentration gradient is incorporated in the Marangoni number given in rel. (2) and it could be evaluated quantitatively in the form of ratio of finite differences \( \frac{\Delta c}{\Delta z} \).

The establishing of the depth \( \Delta z \) represents the main challenge and requires a more substantial comment in order to be solved correctly. The same is valid also for the finite difference of concentration \( c \) which could take naturally a wide range of values related to slag, from those of magnitude of statistical concentration fluctuations (3-5% from the mean value obtained by chemical analysis inclusive the maximal solubility value of sulphur), up to major inhomogeneities, as it was signaled in steel [6] and which could be several hundreds times higher than the bulk concentration of sulphur in steel.

Depending on the system, many other expressions available in literature may be used, but this must be done carefully because wrong selections of representative parameters and of edificatory considered experimental data have been signaled, in analyzing the Marangoni effect based on them.

The model Reichembach-Linde [12] investigates the stability of plane interface using the linear hydrodynamical stability theory and it is was used in evaluation of neutral and oscillatory marginal stability. Values of critical Marangoni dimensionless number from rel. (2), related to upper phase (1), have been computed for marginal states of stationary and oscillatory instabilities. A flat interface between two immiscible fluid layers of finite height was used and a term, expressing the gravity effect (Weber dimensionless number), was introduced as required in the balance of forces on the normal direction in order to obtain critical Marangoni numbers.

Finite depths of the layers in each fluid are taken those where the temperature perturbation vanishes, and it is shown that all considerations and relations from thermocapillarity case and could be used in the solutocapillarity case, respecting the equivalence principles in replacing the involved quantities from thermocapillary case to solutocapillary case. Flatness of interface was introduced in the form of capillary dimensionless number as Crispation number, for which condition \( Cr=0 \) is required.

The resulting set of basic relations and definition of parameters is in the form of the relation presented bellow. Further, the authors used in their numeric simulations relative arbitrarily selected value of ratios of important parameters of the liquid phases forming the interface in question and they did not establish characteristic lengths using the method of scaling, but only a generic unit length \( d \). The intrinsic interfacial viscosity is neglected in the model Reichembach-Linde [13] similarly to in the model Sterling-Sriven [11]. The main relations, quantities and parameters of the model Reichembach-Linde [12], adapted to the mass transfer of sulphur from steel (phase 2) to slag (phase 1) are the following:
\[
Ma_{\text{slag}} = \left(\epsilon + \frac{1}{L_s \cdot D^*}\right) \left(\frac{H_1 + H_2 - \frac{G_1}{\mu^*} \cdot G_2}{1 - E_2}\right) \left(\frac{L_1 + \frac{D^*}{1 + E_2} \cdot G_2}{L_2}\right) \tag{3}
\]

\(Ma_{\text{slag}}\) - critical Marangoni dimensionless number related to slag phase;

\[
D^* = \frac{D_{(S)}}{D_{(S)}} \tag{4}
\]

where: \(D_{(S)}\), \(D_{(S)}\) - is the mass diffusion coefficient of sulphur in the slag and respective in the steel.

\[
\mu^* = \frac{\mu_{\text{slag}}}{\mu_{\text{steel}}} \tag{5}
\]

where: \(\mu_{\text{slag}}, \mu_{\text{steel}}\) - is the dynamic viscosity of the slag and respective of the steel.

\[
L_{(S)} = \frac{(%S)}{[%S]} \tag{6}
\]

is the instant partition ratio of sulphur, between slag[%mass] and steel[%mass];

\[
\epsilon = \frac{(1 - E_2)(1 + E_1)}{(1 + E_2)(1 - E_1)} \tag{7}
\]

\(a = k \cdot d\) - is the dimensionless wave number; \(d\) - is a characteristic length, in [m]; \(k\) - is the wave number, in [m⁻¹]; the wavelength is \(\lambda = 2\pi/k\), in [m];

\[
l_i = \frac{h_i}{d} \tag{11}
\]

\(h_i\) - is the thickness of layer \(i\) (1,2), in [m];

\[
E_i = e^{-2a(l_i)} \tag{12}
\]

\[
H_i = 8a^2 \left(1 - E_i^2 - 4a^2l_i^2E_i\right) \tag{14}
\]

\[
G_i = -(l - E_i)^2 + 4a^2l_i^2E_i \tag{15}
\]

\[
L_i = (1 - E_i)^3 - 4a^3l_i^3E_i(1 + E_i) \tag{16}
\]

Relations (7), (14)-(16) are computational parameters obtained by grouping, with no physical meaning, but useful for easiness of computations.

The model [12] contains mathematical expressions based on exponential functions in base \(e\) (Euler number \(e \approx 2.71828\)) and due to this, the Marangoni dimensionless number according to the relation(3) never could reach the value zero and the graphic of \(Ma=\tilde{h}(k)\) never could intersect the axis of abscise containing the wave number in any form (explicit or implicit variable). Therefore the graphic, either is over abscise, or below it. This means the Marangoni number reaches only a strictly positive value or a strictly negative one.

Also, it is shown in many basic papers that if an asymptote to the graphic would appear, than the development of a possible instability will change from short waves stationary instability to an oscillatory instability even in the form of dilatational waves. The results found in the present paper based on the model [12] show that such zone was found.

For the first time the model [12] was adapted in the present paper to the particular interface LCAK steel-(CaO-Al2O3) slag at temperature 1873.15K. From the whole configuration of the technological system steel-slag, which is extended at infinity, in rectangular coordinates, a small portion will be considered, this having the height of layers according to the preliminary scaling of the characteristic heights and length. This is a first difference in comparison with the approach used in the basic model [12], where arbitrary dimensions and their ratios have been considered. In the present paper all ratios containing quantities \(h, d\) and their ratios, involved in the model, have fixed values, because of the real case considered. In what it follows the adapted model from the present paper is used to evaluate conditions of stability/instability of the mentioned interface in flatness conditions under other particular conditions which are imposed either by the specific values of parameters of phases, or due to the specific scaling which is used. Careful scaling of specific and characteristic lengths and dimensions involved in the present analysis is an important key for a correct approach of the problem, even from its starting point, especially as this has not been done before from various reasons, in many other previous analyzes. Many papers report theoretical analyzes, considerations presented as pertinent, experimental and modeling results, in various liquid systems, which present a certain degree of similarity with the system steel-slag, considered in the present paper. There are different characteristic lengths, specific to each dominant action, therefore the analysis of stability/instability at interface steel-slag and of the neighboring regions in each phase must be performed in known conditions, acceptable physically for all physical and physicochemical competing actions. Scaling had in view conditions ensuring that the Marangoni effect will be the dominant action, among the other physical and physicochemical competing actions, in such high degree prevalence that it will be possible to be directly evaluated as state.

3. Evaluation of marginal stability at interface in established conditions

3.1. Establishing relevant values of parameters involved in the adapted model

Preliminary testing of the model to predict conditions of occurrence stability/instability of the flat interface LCAK steel-(CaO-Al2O3) slag is
performed for the following conditions exposed in the Table 1.

The layer thicknesses \((h_1,h_2)\) selected for computations, in the present paper, are slightly higher than the values given in the Table 1. This represent an acceptable truncation of the boundary layer, used frequently in large open systems in nature and the manner to treat similar problems could be validated also in systems at low dimensional scales, like the system in question in the present problem.

Fully respecting the established notations and their meanings, the selected values are respectively \(h_1=400\times10^{-6}\) m on the slag side, \(h_2=50\times10^{-6}\) m on the steel side and \(d=100\times10^{-6}\) m along the interface slag-steel, related to the slag side, with double function. Thus, in the acceptance of the present paper \(d\) represents the characteristic length, according to the definition given initially [12], but also represents a distance, along the interface, on which the perturbation of surface tension, due to perturbation of interfacial concentration of sulphur related to the slag side is enough strong, continual and persistent, to produce Marangoni flow and convection. Computations of edifying dimensionless numbers for values of interfacial tension in the range of sulphur content in slag (0-2.92mass%S and \(\sigma_i=1.235-1.285\) N/m \(^2\)) and gravitational acceleration \(g=9.81\) m/s\(^2\) give the following results:

- Bond number at interface on the slag side:

\[
Bo=\frac{\rho_{slag} \cdot g \cdot d^2}{\sigma_i} = (2.05-2.13) \times 10^{-4} \quad (17)
\]

Bo number is used instead of Weber dimensionless number.

- Crispation number representing the inverse of capillary number:

\[
Cr=\frac{\mu \cdot D_i}{\sigma_i \cdot d} = (2.29-2.38) \times 10^{-7} \quad (18)
\]

The values express that interfacial tension prevails very strong over gravity (\(Bo<1\) for simply prevailing) and the necessary required condition for flatness of interface (\(Cr<10^{-5}\)) is accomplished. More, in the case of a small perturbation of interfacial tension slag – steel \(\Delta\sigma\) due to an inhomogeneity of sulphur content in slag \(\Delta(S)\), the condition (\(Bo<1\)) still is satisfied if \(\Delta(S)>0.01\) mass%, using a modified Bo dimensionless number, containing at denominator \(\Delta\sigma\), instead of \(\sigma_i\) in the rel. (17).

The parameters considered in the present paper, referring to the considered slag-steel system and the adapted model, fully satisfy the set of conditions of the initial model Reichembach-Linde[12].

Following values of computational parameters in the adapted model have been established using data from Table 1.

\[
\frac{h_1}{d} = 8; \quad \frac{h_2}{d} = 0.5 \quad (19)
\]

**Table 1.** Physico-chemical quantities and parameters of (60%)CaO-(40%)Al\(_2\)O\(_3\) slags and LCAK steels, at the temperature 1873K

<table>
<thead>
<tr>
<th>Quantity, symbol, units</th>
<th>(1) Slag</th>
<th>(2) Steel</th>
<th>phases, values, references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, (\rho), kg/m(^3)</td>
<td>2685 [13]</td>
<td>7000 [14]</td>
<td></td>
</tr>
<tr>
<td>Dynamic viscosity, (\mu), Pa(\cdot)s</td>
<td>0.1184 [13]</td>
<td>0.007 [14]</td>
<td>4.41 \times 10^{-6}</td>
</tr>
<tr>
<td>Kinematic viscosity, (\nu), m(^2)/s</td>
<td>4.41 \times 10^{-6}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Dimensionless number, (\eta^*)</td>
<td>(\eta^* = \eta_1/\eta_2 = 16.914)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diffusion coefficient of sulphur (D_s), m(^2)/s</td>
<td>2.48 \times 10^{-10} [16]</td>
<td>4.4 \times 10^{-9} [14]</td>
<td></td>
</tr>
<tr>
<td>Solubility of sulphur, %mass</td>
<td>2.092 [17]</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Concentration coefficient of interfacial tension , (\frac{\partial\sigma_i}{\partial(%S)})</td>
<td>-23.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boundary layer thickness, based on concept of energy dissipation in the boundary layer, (h_1), x10(^6) m</td>
<td>355.76 (**)</td>
<td>49.021 (**)</td>
<td></td>
</tr>
<tr>
<td>- average of values related to sulphur content in slag and using relation: (\sigma_i = 0.7353(%S)^2 - 25.504(%S) + 1284.8) [9]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D^2 = D_1/D_2 = 56.36 \times 10^{-3})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) obtained using rel. given in [18];
(**) obtained using rel. given in [19]

**3.2. Computational results**

The results of computation using the adapted model, the values of ratios of involved quantities (Table 1) and the parameters given in the rel. (19) are presented in the fig. 1 and fig.2 where the specific dispersion relation of the critical values of Marangoni factor Ma’ are given in the form Ma’=f(a) (a-dimensionless wave number, rel. 8), because of the smaller values on the abscise.

The critical Marangoni factor Ma’ is derived from rel. (3) and represents the part of the critical Marangoni dimensionless number (rel.3), depending purely of geometrical configuration of the interface and the elements derived from expanding the perturbation in Fourier components (normal modes).

\[
Ma’=Ma/F \quad (20)
\]

The factor \(F\) contains the influence of mass transfer and partition ratio of sulphur:

\[
F = \left[\varepsilon + \frac{1}{L_S \cdot D^*}\right] \quad (21)
\]
The magnitude of the factor $F$ also depends on the dimensionless number $a$ because of $\varepsilon$ (rel.7) and according to the figure 3 it has slight dependence on values of $a$, but represents the main part of factor $F$ in real technical situation because $L_S > \varepsilon$. Therefore the fraction in rel.(21) has negligible value compared to value of $\varepsilon$.

The graphic of function is below the abscise up to a point of a value $a_0 = 2.881255 \pm 5 \cdot 10^{-6}$, not belonging to the domain of values of the dimensionless wave number, that means negative critical Marangoni numbers. Further, this means that in this range no instability occurs when sulphur is transferred from steel to slag through interface, fully in agreement with case presented by Sterlings and Scriven [11] but valid in case of interface of infinite depths layers. Further, the numerical evolution of dispersion relation $Ma_c=f(a)$ is extremely interesting and it is shown in this paper for the first time.

The most important part and in the same time a contribution revealed in the present paper, to these specific problems, consists in the fact that further, overcoming the value $a_0$, values of $Ma_c$ become positive and evolve by positive values to an asymptote which was established numerically in this case at the value $a_\infty = 3.0058 \pm 3.006$. A better resolution in positioning the asymptote was not possible based on the computations technique used, though other three decimals have been computed. However, the results are very good because the values of $a_\infty$ give by conversion, using rel. (8)-(10), an extremely narrow interval of the wavelength value, $\lambda = (209.021 - 209.035) \cdot 10^{-6}$ m. An exact finite value could not be obtained because of the exponential functions based on rel.(8).

The values of $Ma_c$ evolve in the interval $0 < Ma_c < \infty$ upon values of dimensionless wave number $a$, $a_0 < a < a_\infty$ and represents limits of marginal state of stationary and/or oscillatory instabilities.

From the fig.2 an interval of values $a = 2.883 - 3.0$ results, where $Ma' = 1.667 - 1662$ that lead to moderate values of $Ma_c$ as it is shown in the Fig. 3. These values suggest that interfacial instability of interface steel-slag could occur in the form of short waves, due to the transfer of sulphur from steel to slag.

In the range of dimensionless wave number $a = 2.9 - 3.0$, Marangoni factor $Ma'$ presents a strong linear dependence given by the relation:

$$Ma' = e^{168.69 a^2 - 947.51 a + 1331.7}$$

Presenting the coefficient of determination $R^2 = 1.0$.

Further it results that the values of the critical dimensionless number Marangoni are given by the simple relation:

$$Ma_{c, slag} = F \cdot Ma' = (0.0957a + 0.618 + \frac{1}{L_S \cdot D}) e^{168.69 a^2 - 947.51 a + 1331.7}$$

Fig. 1. Marangoni Factor $Ma'$ function of dimensionless wave number $a$
Fig. 2. Marangoni Factor $Ma'$ (in logarithmic scale) as function of dimensionless wave number $a$

Fig. 3. Values of $\epsilon$ factor, in rel.(3), upon the dimensionless wave number $a$

Rel. (23) is valid in the interval of dimensionless wave number values $a = 2.9 - 3.0$ and values of critical Marangoni number $Ma_c$ are presented in the Table 2 for the system considered here and values of the sulphur partition ratio $L_S \in \{1; 10; 100; 1000\}$ which are frequent in practice of desulphurization under slag.

Table 2. Values of $Ma_{c,\text{slag}}$ in the domain of values $a = 2.9 - 3.0$, at representative values of $L_S$.

<table>
<thead>
<tr>
<th>$L_S$</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>259.053</td>
<td>43.214</td>
<td>21.630</td>
<td>19.471</td>
</tr>
<tr>
<td>2.92</td>
<td>514.364</td>
<td>86.053</td>
<td>43.221</td>
<td>38.938</td>
</tr>
<tr>
<td>2.94</td>
<td>1168.859</td>
<td>196.117</td>
<td>98.843</td>
<td>89.115</td>
</tr>
<tr>
<td>2.96</td>
<td>3039.036</td>
<td>511.543</td>
<td>258.704</td>
<td>233.42</td>
</tr>
<tr>
<td>2.98</td>
<td>9048.512</td>
<td>1527.086</td>
<td>774.943</td>
<td>699.73</td>
</tr>
<tr>
<td>3.0</td>
<td>30824.812</td>
<td>5217.453</td>
<td>2656.717</td>
<td>2400.643</td>
</tr>
</tbody>
</table>
4. Commentaries and conclusions

It was shown that the system LCAK steel-(60%CaO+40%Al2O3) slag at temperature 1873.15 K, during transfer of sulphur from steel to slag could be evaluated from the point of view of stability/instability induced by this transfer. This was done using Reichembach-Linde model [12] which was adapted on the base of physico-chemical parameters of steel and slag. The necessary condition consisting in limited heights of the layers of steel and slag has been realized by truncation method based on the theory of momentum dissipation, using relations that give data in good agreements [18][19].

The results are finally presented in the form of data in table 2 as values of $Ma_{c,slag}=f(a)$ using preliminary results in form of factors $f$ and $Ma'$. These results show that the condition $Ma_{c,slag} > 0$ is accomplished in a small region of values of the dimensionless wave number $a$, $a_0 < a < a_{res}$, $a_0 = 2.881255 \pm 5 \cdot 10^{-6}$ and $a_{res} = 3.0058 - 3.006$. From this region, an even smaller extended region of $a$ values ($a = 2.9 - 3.0$) equivalent to wave lengths $216.6 - 209.44 \mu m$ corresponds to low and moderate values of $Ma_{c,slag} = 20 - 5217$ at $L_e = 10 - 1000$, which seems to be enough easy to be touched and overcome in the technical conditions of desulphurization in current refining processes.

A shorter explanation of this behavior of the system LCAK steel-(60%CaO+40%Al2O3) slag at temperature 1873.15K, could be that sulphur, once being transferred at interface on the slag side, at certain values of its partition ratios $L_e$ because of the resistance to mass transfer by diffusion inside of slag, acts as an interfacial inhomogeneity and produces a perturbation of the surface tension which is relaxed by flow( mass transport) along the interface, if this perturbation is enough high, persistent and continual. In these conditions, current values of $Ma_{slag} > Ma_{c,slag}$ and ratios $Ma_{slag}/ Ma_{c,slag}$ have high values leading to occurrence of Marangoni convection. Stationary instability in the form of short wavelength waves could develop in the mentioned conditions or, considering the asymptote found at $a_0 = 3.0058 - 3.006$ oscillatory waves could develop.

The narrow interval of $a/\lambda$ values, found in this research, where the values of $Ma_{c,slag}$ are positive and relative small, allowing occurrence of instability easy enough, could represents what is called a selection mode of wavelength in Marangoni convection, concept which is not enough clarified and argued, but frequently signaled and presented, often with some restraint and caution.

References