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Statistical Analysis Regarding the Evolutions of the Euro Exchange Rate and the Dollar Exchange Rate, in Romania

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ABSTRACT

This paper reflects a the statistical modeling of the values concerning the annual averages of the euro exchange rate, respectively the dollar exchange rate in Romania, through by means of the „Least Squares Method". The exchange rate represents the price regardinga monetary unit from the currency which belongs to a country, expressed in the monetary unit of the another country. Also, the exchange rate takes into consideration the type of quotation which linking the two currencies involved at the exchange ratio. The exchange rates have been more volatile over time, then relative price levels and rates of inflation.

1. Introduction

This research reflects a statistical analysis of the trends regarding the annual averages of the euro exchange rate, respectively the dollar exchange rate, in Romania, between the years 2000-2014. The purpose of the research reflects the possibility for to anticipate the values concerning the evolutions in future of the annual averages for these exchange rates by means of the forecasting method. The statistical methods used are the „Coefficients of Variation Method", the „Least Squares Method" applied for to calculate the parameters of the regression equation and the Forecasting Method through the „Least Squares Method". The sections 2, respectively the section 3, present the methodology for to achieve the trends models for the annual averages of the euro exchange rate, respectively for the dollar exchange rate, with the help of the „Least Squares Method". The section 4 expresses the forecasting method reflected by the „Least Squares Method" applied for these exchange rates. The state of the art in this domain is represented by the research belongs to Carl Friederich Gauss, who created the „Least Squares Method" [1].

2. The modeling of the trend for the evolution regarding the annual average of the euro exchange rate, in Romania, between 2000-2014

In Romania, in the period 2000-2014, we observe the next evolution concerning the annual average of the euro exchange rate, according to the table no. 1:

Table no. 1 The evolution of the annual average of the euro exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE EURO EXCHANGE RATE IN ROMANIA (RON)	INFLATION RATE (%)	THE ANNUAL AVERAGE OF THE EURO EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON)
2000	1,9955	45,7	7,1693
2001	2,6026	34,5	6,9520
2002	3,1255	22,5	6,8153
2003	3,7555	15,3	7,1024
2004	4,0532	11,9	6,8502
2005	3,6234	9,00	5,6182
2006	3,5245	6,56	5,1284
2007	3,3373	4,84	4,6319
2008	3,6827	7,85	4,7392
2009	4,2373	5,59	5,1642
2010	4,2099	6,09	4,8363
2011	4,2379	5,79	4,6020
2012	4,4560	3,33	4,6829
2013	4,4190	3,98	4,4663
2014	4,4446	1,07	4,4446

Source: „B.N.R"

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We want to identify the trend model for the annual average of the euro exchange rate, in Romania, in the period 2000-2014, using the table no. 1.

- if we formulate the null hypothesis H_0 *:* which mentions the assumption of the existence for the model of tendency of the factor *X = the annual average of the euro exchange rate, in Romania,* as being the function $x_{t_i} = a + b \cdot t_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (x_i - x_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (x_i - a - bt_i)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (x_i - a - bt_i)(-1) = 0 /(-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n} (x_i - a - bt_i)(-t_i) = 0 /(-\dfrac{1}{2}) \end{cases} \Rightarrow \begin{cases} na + b\sum_{i=1}^{n} t_i = \sum_{i=1}^{n} x_i \\ a\sum_{i=1}^{n} t_i + b\sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} x_i t_i \end{cases} \Rightarrow$$

Therefore,

$$a = \frac{\begin{vmatrix} \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} x_i t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} t_i^2 - \sum_{i=1}^{n} x_i t_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2} \quad b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} x_i t_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{n\sum_{i=1}^{n} x_i t_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} x_i}{n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2}$$

Table no. 2 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution of the annual average of the euro exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE EURO EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) (x_i)	*LINEAR TREND*				
		t_i	t_i^2	$t_i x_i$	$x_{t_i} = a + bt_i$	$\lvert x_i - x_{t_i} \rvert$
2000	**7,1693**	-7	49	-50,1851	7,096589994	0,0727
2001	**6,9520**	-6	36	-41,7120	6,875202852	0,0768
2002	**6,8153**	-5	25	-34,0765	6,653815710	0,1615
2003	**7,1024**	-4	16	-28,4084	6,432428568	0,6700
2004	**6,8502**	-3	9	-20,5506	6,211041426	0,6392
2005	**5,6182**	-2	4	-11,2364	5,989654284	0,3715
2006	**5,1284**	-1	1	-5,1284	5,768267142	0,6399
2007	**4,6319**	0	0	0	5,546880000	0,9150
2008	**4,7392**	1	1	4,7392	5,325492858	0,5863
2009	**5,1642**	2	4	10,3284	5,104105716	0,0601
2010	**4,8363**	3	9	14,5089	4,882718574	0,0464
2011	**4,6020**	4	16	18,4080	4,661331432	0,0593
2012	**4,6829**	5	25	23,4145	4,439944290	0,2430
2013	**4,4663**	6	36	26,7978	4,218557148	0,2477
2014	**4,4446**	7	49	31,1122	3,997170006	0,4474
TOTAL	**83,2032**	0	280	-61,9884	83,2032	5,2368

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a = \frac{\begin{vmatrix} 83{,}2032 & 0 \\ -61{,}9884 & 280 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{83{,}2032 \cdot 280 - (-61{,}9884) \cdot 0}{15 \cdot 280 - 0 \cdot 0} = 5{,}54688$$

$$b = \frac{\begin{vmatrix} 15 & 83{,}2032 \\ 0 & -61{,}9884 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{15 \cdot (-61{,}9884) - 0 \cdot 83{,}2032}{15 \cdot 280 - 0 \cdot 0} = -0{,}221387142$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^I \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^I \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{5{,}2368}{83{,}2032} \cdot 100 = 6{,}29\%$$

- in the situation of the alternative hypothesis H_1 *:* which specifies the assumption of the existence for the model of tendency of the factor *X = the annual average of the euro exchange rate, in Romania*, as being the quadratic function $x_{t_i} = a + b \cdot t_i + ct_i^2$, the parameters *a*, *b* şi *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (x_i - x_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (x_i - a - bt_i - ct_i^2)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \\ \dfrac{\partial S}{\partial c} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (x_i - a - bt_i - ct_i^2)(-1) = 0 / (-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n} (x_i - a - bt_i - ct_i^2)(-t_i) = 0 / (-\dfrac{1}{2}) \\ 2\sum (x_i - a - bt_i - ct_i^2)(-t_i^2) = 0 / (-\dfrac{1}{2}) \end{cases} \Rightarrow$$

Therefore,

$$\begin{cases} n \cdot a + b\sum_{i=1}^{n} t_i + c\sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} x_i \\ a\sum_{i=1}^{n} t_i + b \cdot \sum_{i=1}^{n} t_i^2 + c\sum_{i=1}^{n} t_i^3 = \sum_{i=1}^{n} t_i \cdot x_i \\ a \cdot \sum_{i=1}^{n} t_i^2 + b\sum_{i=1}^{n} t_i^3 + c\sum_{i=1}^{n} t_i^4 = \sum_{i=1}^{n} t_i^2 \cdot x_i \end{cases}$$

Table no. 3 The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution of the annual average of the euro exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE EURO EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) *(x_i)*	*PARABOLIC TREND*						
		t_i	t_i^2	t_i^3	t_i^4	$t_i^2 x_i$	$x_{t_i} = a + bt_i + ct_i^2$	$\lvert x_i - x_{t_i} \rvert$
2000	**7,1693**	-7	49	-343	2401	351,2957	7,537138480	0,3678
2001	**6,9520**	-6	36	-216	1296	250,272	7,126944837	0,1749
2002	**6,8153**	-5	25	-125	625	170,3825	6,745798348	0,0695
2003	**7,1024**	-4	16	-64	256	113,6384	6,393699013	0,7087
2004	**6,8502**	-3	9	-27	81	61,6518	6,070646832	0,7796
2005	**5,6182**	-2	4	-8	16	22,4728	5,776641805	0,1584
2006	**5,1284**	-1	1	-1	1	5,1284	5,511683932	0,3833
2007	**4,6319**	0	0	0	0	0	5,275773213	0,6439
2008	**4,7392**	1	1	1	1	4,7392	5,068909648	0,3297
2009	**5,1642**	2	4	8	16	20,6568	4,891093237	0,2731
2010	**4,8363**	3	9	27	81	43,5267	4,742323980	0,0940
2011	**4,6020**	4	16	64	256	73,6320	4,622601877	0,0206
2012	**4,6829**	5	25	125	625	117,0725	4,531926928	0,1510
2013	**4,4663**	6	36	216	1296	160,7868	4,470299133	0,0040
2014	**4,4446**	7	49	343	2401	217,7854	4,437718492	0,0069
TOTAL	**83,2032**	0	280	0	9352	1613,041	83,20319976	4,1654

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 15 \cdot a + 0 \cdot b + 280 \cdot c = 83{,}2032 \\ 0 \cdot a + 280 \cdot b + 0 \cdot c = -61{,}9884 \\ 280 \cdot a + 0 \cdot b + 9352 \cdot c = 1613{,}041 \end{cases} \Rightarrow$$

$$a = 5{,}275773213 \qquad b = -0{,}221387142 \qquad c = 0{,}014523577$$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{4{,}1654}{83{,}2032} \cdot 100 = 5{,}01\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition the assumption of the existence for the model of tendency of the factor *X = the annual average of the euro exchange rate, in Romania*, as being the exponential function $x_{t_i} = ab^{t_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (\lg x_i - \lg x_{t_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\lg x_i - \lg a - t_i \lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n}(\lg x_i - \lg a - t_i \lg b)(-1) = 0/(-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n}(\lg x_i - \lg a - t_i \lg b)(-t_i) = 0/(-\dfrac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n \cdot \lg a + \lg b \cdot \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \lg x_i \\ \lg a \sum_{i=1}^{n} t_i + \lg b \cdot \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} t_i \cdot \lg x_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg x_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i \lg x_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg x_i \sum_{i=1}^{n} t_i^{\,2} - \sum_{i=1}^{n} t_i \lg x_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg x_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i \lg x_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^{\,2} \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} t_i \lg x_i - \sum_{i=1}^{n} \lg x_i \sum_{i=1}^{n} t_i}{n\sum_{i=1}^{n} t_i^{\,2} - \left(\sum_{i=1}^{n} t_i\right)^2}$$

Table no. 4 The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution of the annual average of the euro exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE EURO EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) (x_i)	*EXPONENTIAL TREND*					
		t_i	$\lg x_i$	$t_i \lg x_i$	$\lg x_{ti} = \lg a + t_i \lg b$	$x_{ti} = ab^{t_i}$	$\lvert x_i - x_{t_i} \rvert$
2000	**7,1693**	-7	0,855476753	-5,988337277	0,854937087	7,160396754	0,0089
2001	**6,9520**	-6	0,842109763	-5,052658581	0,838022089	6,886873233	0,0651
2002	**6,8153**	-5	0,833484977	-4,167424888	0,821107091	6,623798172	0,1915
2003	**7,1024**	-4	0,851405127	-3,405620511	0,804192093	6,370772445	0,7316
2004	**6,8502**	-3	0,835703251	-2,507109754	0,787277095	6,127412173	0,7228
2005	**5,6182**	-2	0,749597195	-1,499194391	0,770362097	5,893348139	0,2751
2006	**5,1284**	-1	0,709981891	-0,709981891	0,753447099	5,668225232	0,5398
2007	**4,6319**	0	0,665759174	0	0,736532101	5,451701906	0,8198
2008	**4,7392**	1	0,675705036	0,675705036	0,719617103	5,243449661	0,5042
2009	**5,1642**	2	0,713003053	1,426006107	0,702702105	5,043152546	0,1210
2010	**4,8363**	3	0,684513232	2,053539698	0,685787107	4,850506680	0,0142
2011	**4,6020**	4	0,662946614	2,651786457	0,668872109	4,665219787	0,0632
2012	**4,6829**	5	0,670514883	3,352574419	0,651957111	4,487010760	0,1959
2013	**4,4663**	6	0,649947891	3,899687347	0,635042113	4,315609227	0,1507
2014	**4,4446**	7	0,647832681	4,534828774	0,618127115	4,150755146	0,2938
TOTAL	**83,2032**	0	11,04798152	-4,736199456			4,6976

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 11{,}04798152 & 0 \\ -4{,}736199456 & 280 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{11{,}04798152 \cdot 280 - (-4{,}736199456) \cdot 0}{15 \cdot 280 - 0 \cdot 0} = 0{,}736532101$$

$$\lg b = \frac{\begin{vmatrix} 15 & 11{,}04798152 \\ 0 & -4{,}73699456 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{15 \cdot (-4{,}73699456) - 0 \cdot 11{,}04798152}{15 \cdot 280 - 0 \cdot 0} = -0{,}016914998$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{4{,}6976}{83{,}2032} \cdot 100 = 5{,}65\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend.
We notice that:

$$v_{II} = 5{,}01\% < v_{\exp} = 5{,}65\% < v_I = 6{,}29\%$$

So, the path reflected by X factor, which represents the annual average of the euro excchange rate, in Romania, between 2000-2014, is a parabolical trend of the shape $x_{t_i} = a + b \cdot t_i + ct_i^2$, with other words it confirms the hypothesis H_1.

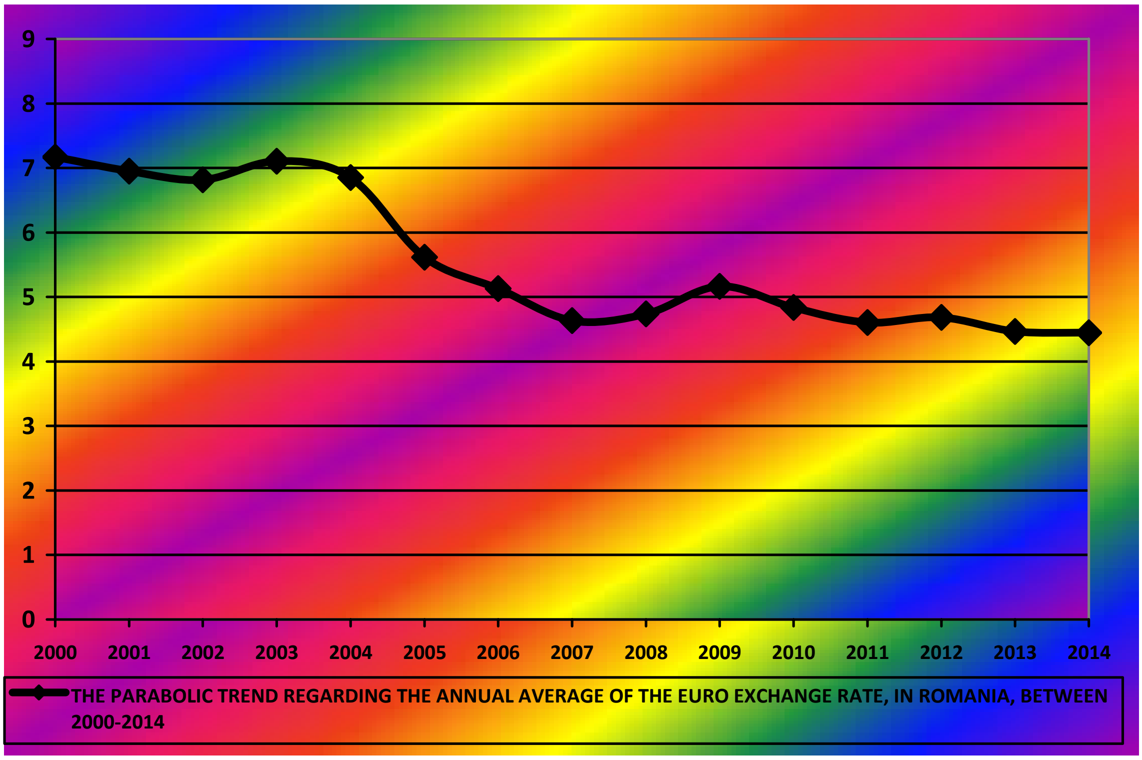

Figure 1. The trend model of the values for the annual average of the euro exchange rate, in Romania, between 2000-2014

We observe that, the cloud of points which reflects the values concerning the annual averages of the euro exchange rate, in Romania, between 2000-2014, it carrying around a parabolical trend model, according to the type no.1.

3. The modeling of the trend for the evolution regarding the annual average of the dollar exchange rate, in Romania, between 2000-2014.

In Romania, in the period 2000-2014, we observe the next evolution concerning the annual average of the dollar exchange rate, according to the table no. 5:

Table no. 5 The evolution of the annual average of the dollar exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE DOLLAR EXCHANGE RATE IN ROMANIA (RON)	INFLATION RATE (%)	THE ANNUAL AVERAGE OF THE DOLLAR EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON)
2000	2,1692	45,7	7,7934
2001	2,9060	34,5	7,7625
2002	3,3055	22,5	7,2078
2003	3,3200	15,3	6,2788
2004	3,2636	11,9	5,5158
2005	3,1078	9,00	4,8188
2006	2,5676	6,56	3,7361
2007	2,4564	4,84	3,4093
2008	2,8342	7,85	3,6473
2009	2,9361	5,59	3,5784
2010	3,2045	6,09	3,6813
2011	3,3393	5,79	3,6262
2012	3,3575	3,33	3,5255
2013	3,2551	3,98	3,2899
2014	3,6868	1,07	3,6868

Source: „B.N.R”

We want to identify the trend model for the average of the dolar cours in a year, in Romania, in the period 2000-2014, using the table no. 5.

- if we formulate the null hypothesis H_0 *:* which mentions the assumption of the existence for the model of tendency of the factor *Y = the annual average of the dollar exchange rate, in Romania,* as being the function $y_{t_i} = a + b \cdot t_i$, then the parameters *a* and *b* of the adjusted linear function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (y_i - y_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (y_i - a - bt_i)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a} = 0 \\ \dfrac{\partial S}{\partial b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum\limits_{1=1}^{n} (y_i - a - bt_i)(-1) = 0 / (-\dfrac{1}{2}) \\ 2\sum\limits_{1=1}^{n} (y_i - a - bt_i)(-t_i) = 0 / (-\dfrac{1}{2}) \end{cases} \Rightarrow \begin{cases} na + b\sum\limits_{i=1}^{n} t_i = \sum\limits_{i=1}^{n} y_i \\ a\sum\limits_{i=1}^{n} t_i + b\sum\limits_{i=1}^{n} t_i^2 = \sum\limits_{i=1}^{n} y_i t_i \end{cases} \Rightarrow$$

Therefore,

$$a = \frac{\begin{vmatrix} \sum\limits_{i=1}^{n} y_i & \sum\limits_{i=1}^{n} t_i \\ \sum\limits_{i=1}^{n} y_i t_i & \sum\limits_{i=1}^{n} t_i^2 \end{vmatrix}}{\begin{vmatrix} n & \sum\limits_{i=1}^{n} t_i \\ \sum\limits_{i=1}^{n} t_i & \sum\limits_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{\sum\limits_{i=1}^{n} y_i \sum\limits_{i=1}^{n} t_i^2 - \sum\limits_{i=1}^{n} y_i t_i \sum\limits_{i=1}^{n} t_i}{n\sum\limits_{i=1}^{n} t_i^2 - \left(\sum\limits_{i=1}^{n} t_i\right)^2} \quad b = \frac{\begin{vmatrix} n & \sum\limits_{i=1}^{n} y_i \\ \sum\limits_{i=1}^{n} t_i & \sum\limits_{i=1}^{n} y_i t_i \end{vmatrix}}{\begin{vmatrix} n & \sum\limits_{i=1}^{n} t_i \\ \sum\limits_{i=1}^{n} t_i & \sum\limits_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{n\sum\limits_{i=1}^{n} y_i t_i - \sum\limits_{i=1}^{n} t_i \sum\limits_{i=1}^{n} y_i}{n\sum\limits_{i=1}^{n} t_i^2 - \left(\sum\limits_{i=1}^{n} t_i\right)^2}$$

Table no. 6 The estimate of the value for the variation coefficient in the case of the adjusted linear function, in the hypothesis concerning the linear evolution of the annual average of the dollar exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE DOLLAR EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) (y_i)	*LINEAR TREND*				
		t_i	t_i^2	$t_i y_i$	$y_{t_i} = a + bt_i$	$\lvert y_i - y_{t_i}\rvert$
2000	**7,7934**	-7	49	-54,5538	7,087446667	0,7060
2001	**7,7625**	-6	36	-46,5750	6,756458093	1,0060
2002	**7,2078**	-5	25	-36,0390	6,425469522	0,7823
2003	**6,2788**	-4	16	-25,1152	6,094480951	0,1843
2004	**5,5158**	-3	9	-16,5474	5,763492380	0,2477
2005	**4,8188**	-2	4	-9,6376	5,432503809	0,6137
2006	**3,7361**	-1	1	-3,7361	5,101515238	1,3654
2007	**3,4093**	0	0	0	4,770526667	1,3612
2008	**3,6473**	1	1	3,6473	4,439538096	0,7922
2009	**3,5784**	2	4	7,1568	4,108549525	0,5301
2010	**3,6813**	3	9	11,0439	3,777560954	0,0963
2011	**3,6262**	4	16	14,5048	3,446572383	2,8204
2012	**3,5255**	5	25	17,6275	3,115583812	0,4099
2013	**3,2899**	6	36	19,7394	2,784595241	0,5053
2014	**3,6868**	7	49	25,8076	2,453606670	1,2332
TOTAL	**71,5579**	0	280	-92,6768	71,55790001	12,654

If we calculate the statistical data for to adjust the linear function, we obtain for the parameters *a* and *b* the values:

$$a = \frac{\begin{vmatrix} 71,5579 & 0 \\ -92,6768 & 280 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{71,5579 \cdot 280 - (-92,6768) \cdot 0}{15 \cdot 280 - 0 \cdot 0} = 4,770526667$$

$$b = \frac{\begin{vmatrix} 15 & 71,5579 \\ 0 & -92,6768 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{15 \cdot (-92,6768) - 0 \cdot 71,5579}{15 \cdot 280 - 0 \cdot 0} = -0,330988571$$

Hence, the coefficient of variation for the adjusted linear function is:

$$v_I = \left[\frac{\sum_{i=-m}^{m} \lvert y_i - y_{t_i}^I \rvert}{n} : \frac{\sum_{i=-m}^{m} y_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \lvert y_i - y_{t_i}^I \rvert}{\sum_{i=-m}^{m} y_i} \cdot 100 = \frac{12,654}{71,5579} \cdot 100 = 17,68\%$$

- in the situation of the alternative hypothesis H_1 *:* which specifies the assumption of the existence for the model of tendency of the factor *Y = the annual average of the dollar exchange rate, in Romania*, as being the quadratic function $y_{t_i} = a + b \cdot t_i + ct_i^2$, the parameters *a*, *b* și *c* of the adjusted quadratic function, can to be calculated by means of the system [1]:

$$S = \sum_{i=1}^{n} (y_i - y_{ti})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (y_i - a - bt_i - ct_i^2)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial a}=0 \\ \dfrac{\partial S}{\partial b}=0 \Rightarrow \\ \dfrac{\partial S}{\partial c}=0 \end{cases} \begin{cases} 2\sum_{1=1}^{n}(y_i-a-bt_i-ct_i^2)(-1)=0/(-\frac{1}{2}) \\ 2\sum_{1=1}^{n}(y_i-a-bt_i-ct_i^2)(-t_i)=0/(-\frac{1}{2}) \Rightarrow \\ 2\sum(y_i-a-bt_i-ct_i^2)(-t_i^2)=0/(-\frac{1}{2}) \end{cases}$$

Therefore,

$$\begin{cases} n\cdot a+b\sum_{i=1}^{n}t_i+c\sum_{i=1}^{n}t_i^2=\sum_{i=1}^{n}y_i \\ a\sum_{i=1}^{n}t_i+b\cdot\sum_{i=1}^{n}t_i^2+c\sum_{i=1}^{n}t_i^3=\sum_{i=1}^{n}t_i\cdot y_i \\ a\cdot\sum_{i=1}^{n}t_i^2+b\sum_{i=1}^{n}t_i^3+c\sum_{i=1}^{n}t_i^4=\sum_{i=1}^{n}t_i^2\cdot y_i \end{cases}$$

Table no. 7 The estimates of the value for the variation coefficient in the case of the adjusted quadratic function, in the hypothesis concerning the parabolic evolution of the annual average of the dollar exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE DOLLAR EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) (y_i)	*PARABOLIC TREND*						
		t_i	t_i^2	t_i^3	t_i^4	$t_i^2 y_i$	$y_{t_i}=a+bt_i+ct_i^2$	$\lvert y_i-y_{t_i}\rvert$
2000	**7,7934**	-7	49	-343	2401	381,8766	8,375882933	0,5825
2001	**7,7625**	-6	36	-216	1296	279,4500	7,492707389	0,2698
2002	**7,2078**	-5	25	-125	625	180,1950	6,694483687	0,5133
2003	**6,2788**	-4	16	-64	256	100,4608	5,981211827	0,2976
2004	**5,5158**	-3	9	-27	81	49,6422	5,352891809	0,1629
2005	**4,8188**	-2	4	-8	16	19,2752	4,809523633	0,0093
2006	**3,7361**	-1	1	-1	1	3,7361	4,351107299	0,6150
2007	**3,4093**	0	0	0	0	0	3,977642807	0,5683
2008	**3,6473**	1	1	1	1	3,6473	3,689130157	0,0418
2009	**3,5784**	2	4	8	16	14,3136	3,485569349	0,0928
2010	**3,6813**	3	9	27	81	33,1317	3,366960383	0,3143
2011	**3,6262**	4	16	64	256	58,0192	3,333303259	0,2929
2012	**3,5255**	5	25	125	625	88,1375	3,384597977	0,1409
2013	**3,2899**	6	36	216	1296	118,4364	3,520844537	0,2309
2014	**3,6868**	7	49	343	2401	180,6532	3,742042939	0,0552
TOTAL	**71,5579**	0	280	0	9352	1510,9748	71,55789999	4,1875

If we calculate the statistical data for to adjust the quadratic function, we obtain for the parameters *a, b* and *c* the next values:

$$\begin{cases} 15\cdot a+0\cdot b+280\cdot c=71{,}5579 \\ 0\cdot a+280\cdot b+0\cdot c=-92{,}6768 \qquad \Rightarrow \\ 280\cdot a+0\cdot b+9352\cdot c=1510{,}9748 \end{cases}$$

$a=3{,}977642807$ $\qquad b=-0{,}330988571$ $\qquad c=0{,}042475921$

So, the coefficient of variation for the adjusted quadratic function has the value:

$$v_{II} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{II} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{4{,}1875}{71{,}5579} \cdot 100 = 5{,}85\%$$

- in the case of the alternative hypothesis H_2 *:* which describes the supposition the assumption of the existence for the model of tendency of the factor *Y = the annual average of the dollar exchange rate, in Romania*, as being the exponential function $y_{t_i} = ab^{t_i}$, then the parameters *a* and *b* of the adjusted exponential function, can to be calculated by means of the next system [1]:

$$S = \sum_{i=1}^{n} (\lg y_i - \lg y_{t_i})^2 = \min \Leftrightarrow S = \sum_{i=1}^{n} (\lg y_i - \lg a - t_i \lg b)^2 = \min$$

$$\begin{cases} \dfrac{\partial S}{\partial \lg a} = 0 \\ \dfrac{\partial S}{\partial \lg b} = 0 \end{cases} \Rightarrow \begin{cases} 2\sum_{1=1}^{n} (\lg y_i - \lg a - t_i \lg b)(-1) = 0 /(-\dfrac{1}{2}) \\ 2\sum_{1=1}^{n} (\lg y_i - \lg a - t_i \lg b)(-t_i) = 0 /(-\dfrac{1}{2}) \end{cases} \Rightarrow$$

$$\begin{cases} n \cdot \lg a + \lg b \cdot \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} \lg y_i \\ \lg a \sum_{i=1}^{n} t_i + \lg b \cdot \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} t_i \cdot \lg y_i \end{cases}$$

Thus,

$$\lg a = \frac{\begin{vmatrix} \sum_{i=1}^{n} \lg y_i & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i \lg y_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{\sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} t_i^2 - \sum_{i=1}^{n} t_i \lg y_i \sum_{i=1}^{n} t_i}{n \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i \right)^2}$$

and

$$\lg b = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} \lg y_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i \lg y_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} t_i \\ \sum_{i=1}^{n} t_i & \sum_{i=1}^{n} t_i^2 \end{vmatrix}} = \frac{n \cdot \sum_{i=1}^{n} t_i \lg y_i - \sum_{i=1}^{n} \lg y_i \sum_{i=1}^{n} t_i}{n \sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i \right)^2}$$

Table no. 8 The estimate of the value for the variation coefficient in the case of the adjusted exponential function, in the hypothesis concerning the exponential evolution of the annual average of the dollar exchange rate, in Romania, between 2000-2014

YEARS	THE ANNUAL AVERAGE OF THE DOLLAR EXCHANGE RATE IN COMPARABLE PRICES OF 2014 YEAR (RON) (y_i	A. EXPONENTIAL TREND					
		t_i	$\lg y_i$	$t_i \lg y_i$	$\lg y_{ti} = \lg a + t_i \lg b$	$y_{ti} = ab^{t_i}$	$\lvert y_i - y_{t_i}\rvert$
2000	**7,7934**	-7	0,891726967	-6,242088770	0,837672286	6,881328427	0,9121
2001	**7,7625**	-6	0,890001613	-5,340009679	0,811664542	6,481336084	1,2812
2002	**7,2078**	-5	0,857802727	-4,289013637	0,785656798	6,104594176	1,1032
2003	**6,2788**	-4	0,797876649	-3,191506598	0,759649054	5,749751219	0,5290
2004	**5,5158**	-3	0,741608510	-2,224825532	0,733641310	5,415534290	0,1003
2005	**4,8188**	-2	0,682938901	-1,365877803	0,707633566	5,100744454	0,2819
2006	**3,7361**	-1	0,572418492	-0,572418492	0,681625822	4,804252469	1,0681
2007	**3,4093**	0	0,532665218	0,532665218	0,655618078	4,524994733	1,1157
2008	**3,6473**	1	0,561971486	0,561971486	0,629610334	4,261969466	0,6147
2009	**3,5784**	2	0,553688885	1,107377770	0,603602590	4,014233122	0,4358
2010	**3,6813**	3	0,566001210	1,698003632	0,577594846	3,780896997	0,0996
2011	**3,6262**	4	0,559451753	2,237807014	0,551587102	3,561124047	0,0651
2012	**3,5255**	5	0,547220719	2,736103595	0,525579358	3,354125883	0,1714
2013	**3,2899**	6	0,517182697	3,103096184	0,499571614	3,159159943	0,1307
2014	**3,6868**	7	0,566649578	3,966547052	0,473563870	2,975526826	0,7113
TOTAL	**71,5579**	0	9,834271182	-7,28216856			8,6201

Consequently, if we calculate the statistical data for to adjust the exponential function, we obtain for the parameters *a* and *b* the values:

$$\lg a = \frac{\begin{vmatrix} 9{,}834271182 & 0 \\ -7{,}28216856 & 280 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{9{,}834271182 \cdot 280 - (-7{,}28216856) \cdot 0}{15 \cdot 280 - 0 \cdot 0} = 0{,}655618078$$

$$\lg b = \frac{\begin{vmatrix} 15 & 9{,}834271182 \\ 0 & -7{,}28216856 \end{vmatrix}}{\begin{vmatrix} 15 & 0 \\ 0 & 280 \end{vmatrix}} = \frac{15 \cdot (-7{,}28216856) - 0 \cdot 9{,}834271182}{15 \cdot 280 - 0 \cdot 0} = -0{,}026007744$$

Accordingly, the coefficient of variation for the adjusted exponential function has the next value:

$$v_{\exp} = \left[\frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{n} : \frac{\sum_{i=-m}^{m} x_i}{n} \right] \cdot 100 = \frac{\sum_{i=-m}^{m} \left| x_i - x_{t_i}^{\exp} \right|}{\sum_{i=-m}^{m} x_i} \cdot 100 = \frac{8{,}6201}{71{,}5579} \cdot 100 = 12{,}05\%$$

We apply the coefficients of variation method as criterion of selection for the best model of trend.
We notice that:

$$v_{II} = 5{,}85\% < v_{\exp} = 12{,}05\% < v_I = 17{,}68\%$$

So, the path reflected by Y factor, which represents the annual average of the dollar exchange rate, in Romania, between 2000-2014, is a parabolical trend of the shape $y_{t_i} = a + b \cdot t_i + ct_i^2$, with other words it confirms the hypothesis H_1.

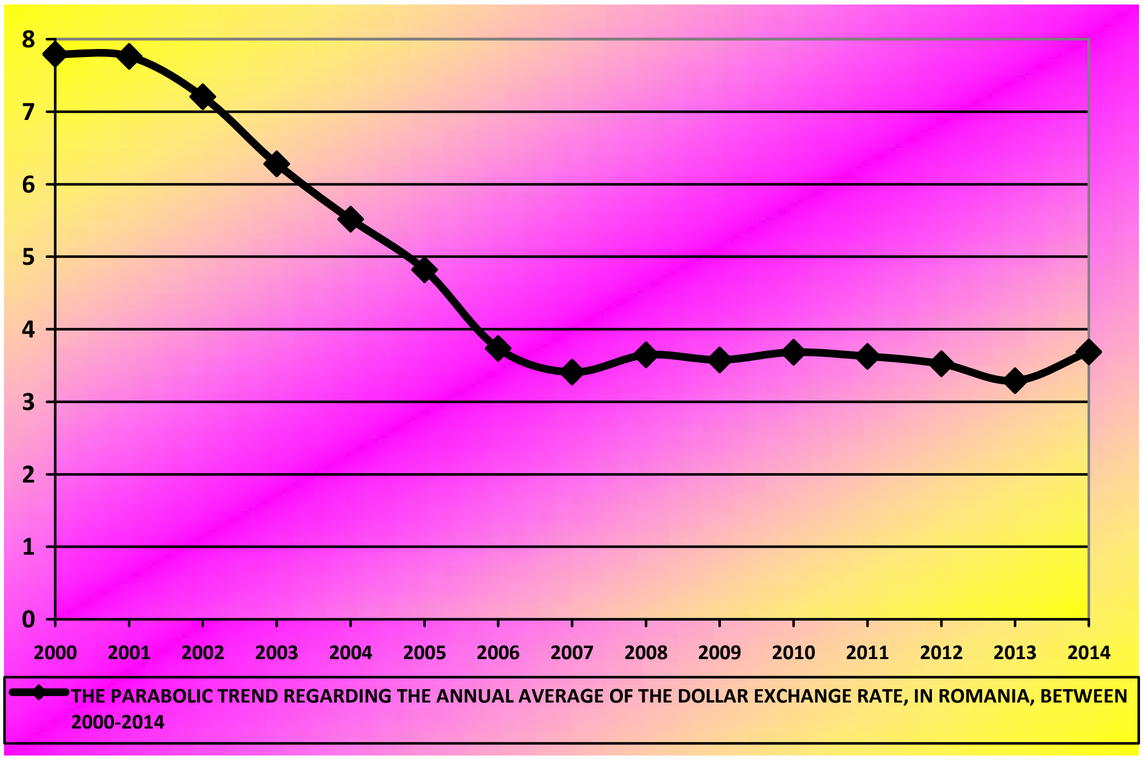

Figure 2. The trend model of the values for the annual average of the dollar exchange rate, in Romania, between 2000-2014

We observe that, the cloud of points which reflects the values regarding the annual averages of the dollar exchange rate, in Romania, between 2000-2014, it carrying around a parabolical trend model, according to the type no. 2.

4. The forecasting method through the „Least Squares Method"

We know that the evolution of the annual average concerning the euro exchange rate, in Romania, between 2000-2014, reflects a parabolic trend of the shape $x_{t_i} = a + b \cdot t_i + ct_i^2$. So, in 2016, the annual average of the euro exchange rate in Romania will be:

$$Euro\_Cours_{2016}^{Romania} = 5,275773213 + (-0,221387142) \cdot 9 + 0,014523577 \cdot 9^2 = 4,459698672 \text{ RON}$$

Also, the trend of the values regarding the annual average of the dollar exchange rate, in Romania, between 2000-2014, is a parabolic trend of the shape $y_{t_i} = a + b \cdot t_i + ct_i^2$. Thus, in 2016, the annual average of the dollar exchange rate in Romania will be:

$$Dolar\_CoursL_{2016}^{Romania} = 3,977642807 + (-0,330988571) \cdot 9 + 0,042475921 \cdot 9^2 = 4,439295269 \text{ RON}$$

4. Conclusions

We can to synthesize that in Romania, the evolutions regarding the annual averages of the euro exchange rate, respectively the dollar exchange rate are growing in 2016. The exchange rate is the price through a currency it changes with another, respectively the ratio between a national currency and one foreign currency.

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