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FAST ACTING ROBUST CONTROLLER FOR PITCH CONTROL SYSTEM OF AIRCRAFT

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Abstract: In this paper a classical feedback controller has been designed to furnish fast acting characteristics in the face of parametric perturbation using two parameters, Generalized Time Constant and Characteristic Ratios. The design is extended to develop a controller for the pitch control of a BRAVO fighter aircraft. This design allows reduction of overshoot, control of speed of the response and robust stability with parametric perturbation.

Keywords: characteristic ratios, generalized time constant, Butterworth polynomial, overshoot reduction.

1. INTRODUCTION

Good transient behavior (Daekwan, et *al.*, 2003; Ali and Burghart, 1991) of a dynamic system in time domain results early settling time and low overshoot. Slow transient response in many cases adversely affects the dynamics of the fast acting system such as aircraft pitch control system of an aircraft. If the pitch angle of an aircraft does not settle to its reference value early, it will affect the longitudinal motion of the aircraft making it difficult for the pilot to maneuver. The fast acting robust controller (Deodher, *et al.,* 1992) tremendously improves the dynamics of the aircraft. The robust stability is tested here using Kharitonov's Stability Criteria (Kharitonov, 1979;.Minnichelli.*et al.*, 1989). The idea is based on the relationship between characteristic polynomial coefficients and time domain response which was initially presented by Naslin (Naslin, 1965; Naslin 1969) in mid 1960.He observed empirically that the step response of all poles of various systems remain unchanged provided the coefficient of characteristic polynomial satisfy certain relation. In 1978 Lipatov and Sokolov (Lipatov and Sokolov, 1979) gave several set of sufficient set of conditions for stability and instability in terms of coefficients of characteristic polynomials. Coincidentally the coefficient relationship used by Naslin to study the transient response happens to be identical with that derived by Lipatov and Sokolov. Thus characteristic ratios of a system are an important parameter for stability and transient response control. Manabe (Manabe, 1998) investigated good transient response of the system with help of characteristic ratios. He also focused on the generic behavior of the plant in the context of Coefficient Diagram Method (CDM) (Manabe, 2002; Ocal and Soylemez, 2005) designing controller for many industrial applications.

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In the same manner Characteristic Ratio Assignment (CRA) (Kim, *et al.*,2003) method is developed to directly address the transient control problem such as overshoot and settling time.

2. CHARACTERISTIC RATIOS

The characteristic polynomials have important properties related to the step response of Linear Time Invariant (LTI) systems.
Let *p(s)* be a polynomial with positive real coefficients as follows

$$(1)\ p(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s^1 + a_0$$

The characteristic ratios as obtained by Naslin (Naslin, 1965a, 1969b)

$$(2)\ \alpha_1 = \frac{a_1^2}{a_0 a_2} \ldots\ldots, \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2}a_n}$$

The characteristic pulstances (Kim and Keel, 2002)

$$(3)\ \beta_0 = \frac{a_0}{a_1}, \beta_0 = \frac{a_1}{a_2}, \ \ldots\ \beta_{n-1} = \frac{a_{n-1}}{a_n}$$

From (2) and (3)

$$(4)\ \alpha_1 = \frac{\beta_1}{\beta_0}, \alpha_1 = \frac{\beta_2}{\beta_1}, \ldots\ \alpha_{n-1} = \frac{\beta_{n-1}}{\beta_{n-2}}$$

The coefficients of polynomial may be represented as

$$(5)\ a_1 = \frac{a_0}{\beta_0}$$

$$(6)\ a_i = \frac{a_0}{\alpha_{i-1}\alpha^2_{i-2}\alpha^3_{i-3}\ldots\alpha_1^{i-1}\beta_0^{\,i}}$$

where $i = 2, 3$

3. GENERALIZED TIME CONSTANT

The time constant of a first order system determines the speed of the response. The time constant is unknown when multiple time constants are present in a system. Due to the unknown relation between multiple time constants and time response it is difficult to achieve desired time response when a higher order transfer function (TF) is involved. So the concept of *generalized time constant* τ (Kim and Keel, 2002) is introduced which precisely relates to the speed of the response.

$$(7)\ \tau = \frac{a_1}{a_0}$$

Let us consider two polynomials *p₁(s)* and *p₂(s)* to construct two all pole TFs *G₁(s)* and *G₂(s)* shown below.

$$G_1(S) = \frac{a_0}{p_1(s)} = \frac{a_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0}$$

$$G_2(S) = \frac{b_0}{p_2(s)} = \frac{b_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_0}$$

for $a_i, b_i > 0$, then $\tau_1 = \frac{a_1}{a_0}$ and $\tau_2 = \frac{b_1}{b_0}$

Theorem 1

Let $y_i(t)$ be the zero state response of $G_i(s)$, i=1,2 to an arbitrary input. Then $y_1(t) = y_2(\frac{\tau_1}{\tau_2}t), \forall\ t \geq 0$.

If and only if both $p_1(s)$ and $p_2(s)$ have the same characteristic ratios:

$$\frac{a_i^2}{a_{i-1}a_{i+1}} = \frac{b_i^2}{b_{i-1}b_{i+1}} = \alpha_i, for\ i = 1, 2, \ldots, n-1$$

The theorem states that the speed of response of a linear all pole system can be controlled while maintaining the exact shape of the response by adjusting the value of τ and keeping its characteristic ratios same. The following example illustrates the theorem.

Example 1

Let $G_i(S) = \frac{a_0}{p_i(s)} = \frac{a_0}{a_6 s^6 + a_5 s^5 + \ldots a_2 s^2 + a_1 s + a_0}$ be an arbitrary 6th order TF for *i =1,2,3.. ..*with

$$p_1(s) = 1.25s^6 + 12.2s^5 + 31.2s^4 + 75.23s^3 + 50s^2 + 12.50s + 5$$

The generalized time constant

$$\tau_1 = \frac{a_1}{a_0} = \frac{12.5}{2.5} = 2.5$$

The characteristic ratios, obtained using equation 2 are $[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5] = [0.625,\ 2.658, 3.627, 1.060,\ 3.816]$

Let *p₂(s), p₃(s)* are two polynomials whose characteristic ratios are same as *p₁(s)* but with different time constants τ_1 and τ_2 . They are constructed using equation 5 and 6 with $\tau_2 = 2$ and $\tau_3 = 1.5$ as follows.

$$p_2(s) = 0.3277\,s^6 + 3.998\,s^5 + 12.78\,s^4 + 38.52\,s^3 + 32\,s^2 + 10\,s + 5$$

$$p_3(s) = 0.05832\,s^6 + 0.9487\,s^5 + 4.044\,s^4 + 16.25\,s^3 + 18\,s^2 + 7.5\,s + 5$$

The step response of *G₁(s),G₂(s)* and *G₃(s)* are plotted in fig.1 below. It is observed that the smaller generalized time constant results slower response without altering the shape of it.

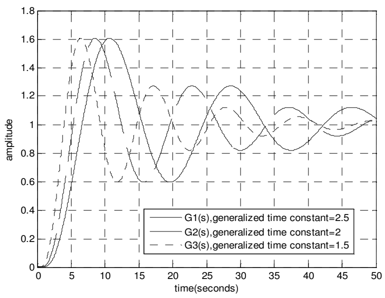

Fig.1. The Step Responses with different Time Constants

4. CHANGING THE SPEED OF THE RESPONSE (GENERALIZED TIME CONSTANT METHOD)

For a given all pole system with a given generalized time constant Theorem 1 can be extended to determine a new generalized time constant which will provide the desired speed of the response while maintaining the exact shape of the response.

Corollary 1

Suppose that $G_1(s)$ and $G_2(s)$ are all pole systems with same characteristic ratios .Let τ_1 and τ_2 be their respective generalized time constants. Then for an arbitrary t_1 and t_2 $y_1(t_1) = y_2(t_2)$ if and only if

$$\tau_2 = \frac{t_2}{t_1}\tau_1$$

Example 2

Let $G_1(s)$ be an arbitrary all pole TF

$$G_1(s) = \frac{15}{s^5 + 4\,s^4 + 12.1\,s^3 + 24.14\,s^2 + 16\,s + 15}$$

The generalized time constant

$$\tau_1 = \frac{a_1}{a_0} = 1.0667$$

The characteristic polynomials calculated from equation 2 are as follows

$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0.707,\ 3.008,\ 1.5166,\ 1.322]$ The step response $y_1(t)$ of $G_1(s)$ plotted in fig. 2 is shown below. The rise time (where the step response value is 0.9) of $y_1(t)$ is found out to be 2.37 seconds(t_1). Let us find out a TF $G_2(s) = \frac{a_0}{p_2(s)}$ for a faster response having rise time 1 second (t_2) .Then the generalized time constant τ_2 is determined from *Corollary-1* as follows,

$$\tau_2 = \frac{t_2}{t_1}\tau_1 = \frac{1}{2.37}\,(1.0667) = 0.4501$$

$p_2(s)$ is found out using equation .5and 6 as given below

$$p_2(s) = 0.01337\,s^5 + 0.1268\,s^4 + 0.909\,s^3 + 4.297\,s^2 + 6.751\,s + 15$$

$$G_2(s) = \frac{a_0}{p_2(s)} = \frac{15}{0.01337\,s^5 + 0.1268\,s^4 + 0.909\,s^3 + 4.297\,s^2 + 6.751\,s + 15}$$

The TF $G_2(s)$ obtained above produces a faster response. Similarly it is also possible to obtain a TF $G_3(s)$ from *Corollary-1* which will provide the slower response with rise time 4.74 seconds (t_3).

$$G_3(s) = \frac{a_0}{p_3(s)},\ \tau_3 = \frac{t_3}{t_1}\tau_1 = 2.133$$

$p_3(s)$ is found out using equation 5 and 6,

$$p_3(s) = 32\,s^5 + 64\,s^4 + 96.8\,s^3 + 96.54\,s^2 + 32\,s + 15$$

$$G_3(s) = \frac{a_0}{p_3(s)} = \frac{15}{32\,s^5 + 64\,s^4 + 96.8\,s^3 + 96.54\,s^2 + 32\,s + 15}$$

The step responses $y_1(t)$,$y_2(t)$ and $y_3(t)$ of $G_1(s)$,$G_2(s)$ and $G_3(s)$ respectively are plotted in the fig. 2 below.

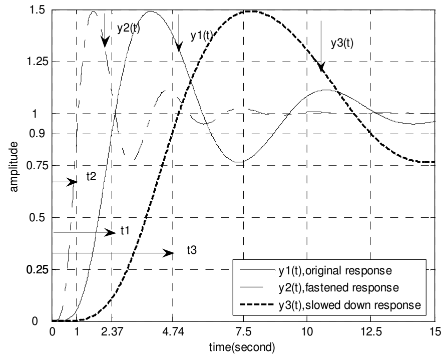

Fig.2. The Unit Step Responses of $y_1(t)$, $y_2(t)$ and $y_3(t)$

As τ varies the poles of system with same characteristic ratios moves along a straight line drawn from the origin shown in the fig.3 below.

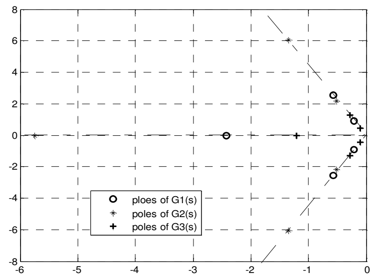

Fig.3. Poles and Zeros of $G_1(s)$,$G_2(s)$,$G_3(s)$

5. CHANGING THE SPEED OF THE RESPONSE (TIME SCALING METHOD)

Let $G(s)$ be the close loop TF which represents the ratio of the out put $Y(s)$ and a step input $R(s)$. If $y(t)$ is the response due to the input $r(t)$,then the response can be speeded up by making $y(t)$ as $y(\beta t)$ where $\beta > 1$. With similar argument the same response can be slowed down if $0<\beta<1$.It is desired to determine a modified system with TF $H(s)$ so that its forced response due to $r(t)$ is $y(\beta t)$ for a given value of β. If Laplace transformation of a time domain function $x(t)$ be $X(s) : x(t) \rightarrow X(s)$, then, $x(\beta t) \rightarrow (1/\beta)\,X(s/\beta)$, where '$\beta$' is a constant. If overshoot remains same only acceleration in time domain is considered then, $x(\beta t) \rightarrow X(s/\beta)$ .So $H(s)$ is the modified TF which produces the speeded or slowed response and is obtained by replacing s by s/β in $G(s)$.

Example 3

This example illustrates how the time response can be speeded up or slowed down without changing the overshoot of an arbitrary 5^{th} order plant

Let $G(s) = \dfrac{15.645}{s^5 + 4s^4 + 12.1s^3 + 24.135s^2 + 21.6\,s + 15.645}$

$$\tau = \frac{a_1}{a_0} = 1.38$$

The characteristic ratios calculated using equation 2 are as follows.

$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [1.2356, 2.2287, 1.5166, 1.3223]$

The step response of the TF *G(s)* is plotted in fig. 4 shown below. The rise time (t_1) of response $y_1(t)$ is found to be 2.40 seconds. This rise time can be decreased by choosing the suitable value of β to fasten the response $y_1(t)$. The response is now decided to be speeded up with rise time (t_2) equals to 1.0 second. So the speed up a factor β ($\beta = t_1/t_2$) reduces to 2.4 seconds. The modified TF $H_1(s)$ is obtained by replacing s by s/β in *G(s)*

$$H_1(s) = \frac{15.645}{0.012s^5 + 0.120s^4 + 0.875s^3 + 4.190s^2 + 9.0s + 15.645}$$

The step response $y_2(t)$ of TF $H_1(s)$ is plotted in Fig. 4 below shows that the rise time is precisely 1.0 second. The rise time (t_1) of the response $y_1(t)$ can be increased to make the system slower if the value of β is less than 1.The response is now slowed down with rise time (t_3) equals to 4.8 seconds resulting the speed up a factor β ($\beta = t_1/t_3$) equals to 0.5. The TF $H_2(s)$ given below is obtained by modifying *G(s)* as before to produces the slowed response.

$$H_2(s) = \frac{15.645}{32.0s^5 + 64.0s^4 + 96.8s^3 + 96.54s^2 + 43.2s + 15.645}$$

The step response $y_3(t)$ of TF $H_2(s)$ is plotted in fig. 4 below shows that the rise time is precisely 4.82 seconds.

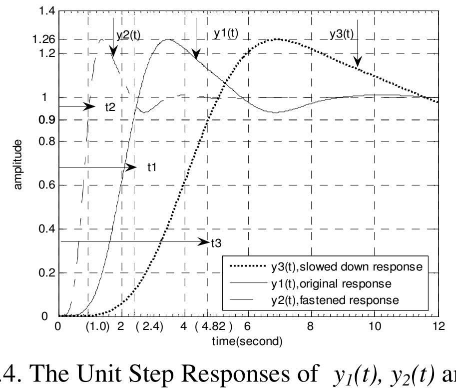

Fig.4. The Unit Step Responses of $y_1(t)$, $y_2(t)$ and $y_3(t)$

6. THE CHARACTERISTIC RATIOS WITH ADJUSTABLE DAMPING

In this section the overshoot of the step response is controlled by relating the multiple characteristic ratios of the system with Butterworth polynomial whose frequency response is monotonically decreasing. Naslin (Kim and Keel, 2002) studied the families of polynomials of varying degree with same first characteristic pulstances β_0 and the same damping factor α defined by equal characteristic ratios as

$$(8)\ \alpha = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_{n-1}$$

With β_0 known the remaining pulsatances can be obtained from equation 4

$$(9)\ \beta_0, \beta_1 = \alpha\beta_0, \beta_2 = \alpha^2\beta_0, \beta_{n-1} = \alpha^{n-1}\beta_0$$

With $a_n=1$,the n^{th} order polynomial can be determined using equation 3 as follows

$$(10)\ p_n(s) = s^n + \alpha^{n-1}\beta_0 s^{n-1} + \alpha^{2n-3}\beta_0^2 s^{n-2} + ...$$

For $n=3,4,5,6$ and 7 a family of polynomials $p_n(s)$ are obtained from equation (10)

(11)

$$\begin{cases} p_2(s) = s^2 + \alpha\beta_0 s + \alpha\beta_0^2 \\ p_3(s) = s^3 + \alpha^2\beta_0 s^2 + \alpha^3\beta_0^2 s + \alpha^3\beta_0^3 \\ p_4(s) = s^4 + \alpha^3\beta_0 s^3 + \alpha^5\beta_0^2 s^2 + \alpha^6\beta_0^3 s^3 + \alpha^6\beta_0^4 \\ p_5(s) = s^5 + \alpha^4\beta_0 s^4 + \alpha^7\beta_0^2 s^2 + \alpha^9\beta_0^3 s^2 + \\ \qquad \alpha^{10}\beta_0^4 s + \alpha^{10}\beta_0^5 \\ p_6(s) = s^6 + \alpha^5\beta_0 s^5 + \alpha^9\beta_0^2 s^4 + \alpha^{12}\beta_0^3 s^3 + \alpha^{14}\beta_0^4 s^2 \\ \qquad + \alpha^{15}\beta_0^5 s + \alpha^{15}\beta_0^6 \\ p_7(s) = s^7 + \alpha^6\beta_0 s^6 + \alpha^{11}\beta_0^2 s^5 + \alpha^{15}\beta_0^3 s^4 + \alpha^{18}\beta_0^4 s^3 \\ \qquad + \alpha^{20}\beta_0^5 s^2 + \alpha^{21}\beta_0^6 s + \alpha^{21}\beta_0^7 \end{cases}$$

With the polynomials shown above for different values of $n=2,4,5,7$ the TFs are constructed as $G_i(s) = \dfrac{a_0}{p_i(s)}$ where a_0 is the constant in $p_i(s)$ and $i=2,4,5,7$ .The unit step responses of all $G_i(s)$ are plotted for different values of α (keeping β constant) to study the effect of damping on responses.

Example 4

For $\alpha=1.5$ and $\beta=1$ the step responses are plotted below in Fig. 5 shows that for $n=2$ the step response is clearly different than the responses obtained from higher order of the polynomials ($n=4,5,7$) .

For $\alpha=2.5$ and $\beta=1$ the step responses plotted below in Fig. 6 shows that responses obtained from all the polynomials ($n=2,4,5,7$) are almost same and difficult to distinguish. With increasing value of α the responses are almost similar and independent of order of the polynomials.

It is noted in this section when α is more than 2 the responses do not dependent on the order of the polynomials. At α =2.5 the responses are free of overshoot which was confirmed from the result of Manabe. A systematic and analytical method is developed in the next section to obtain all pole system with no overshoot.

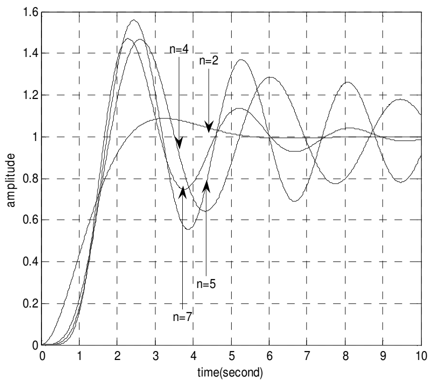

Fig.5. Step Response for $\alpha = 1.5, \beta = 1$

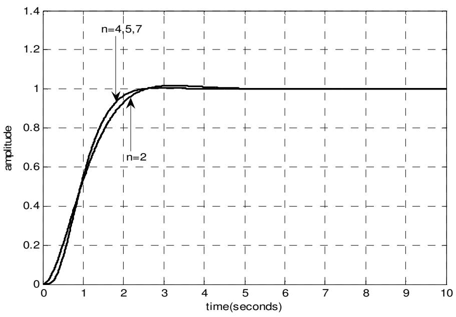

Fig.6. Step Response for $\alpha = 2.5, \beta = 1$

7. STRATEGY FOR NO OVERSHOOT

Chestnut (Kim and Keel, 2002) provided a set of empirical results through a performance chart relating frequency response of the linear system with it’s transient behavior. A function whose frequency magnitude is flat and can control the overshoot with steepness of attenuation slope in high frequency region.
In filter design the magnitude squared response of an analog low pass Butterworth filter $H_a(s)$ of order N is given by (Mitra, 2005)

$$(12)\ |Ha(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

where Ω_c is 3db cutoff frequency.
The maximally flat characteristics are obtained by setting the first *2N-1 derivatives of* $|Ha(j\Omega)|^2\big|_{\Omega=0}$ to zero. So the Butterworth filter is said to have *maximally flat magnitude* at Ω=0 .This maximally flat characteristic is exploited later in this section to construct a Butterworth polynomial to obtain no overshoot.

A Butterworth polynomial is expressed as

$$(13)\ p(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_2 s^2 + a_1 s^1 + a_0$$

The fig. 7 depicts the characteristic of 4th ,6th and 8th order Butterworth filters with 3 dB cutoff frequency at Ω=1 .

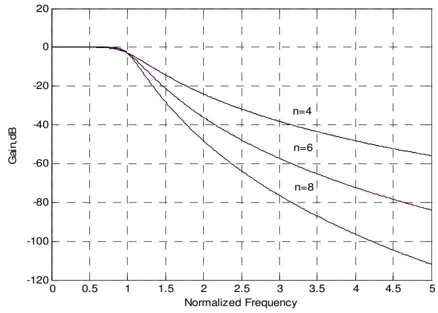

Fig.7. Frequency Magnitude of Butterworth Filters

Next the conditions for obtaining a Butterworth polynomial with help of characteristic ratios (α_i) are discussed with help of the following theorem (Kim, *et al.,* 2003).

Theorem 3
Let G(s) be all pole TF:

$$G(s) = \frac{a_0}{p(s)} = \frac{a_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}, a_i > 0$$

and α_i *, be the characteristic ratios of p(s) .*
A) Then the frequency magnitude function $|G(j\omega)|$ *monotonically decreasing if the following two conditions are held*
B) p(s) is Hurwitz if the following two conditions are met
Conditions:
(1) α_1 >2

$$(2)\ \alpha_k = \frac{\sin\left(\frac{k\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)}{2\sin\left(\frac{k\pi}{n}\right)}.\alpha_1 \quad \textit{for k=2,3,n-1}$$

The above theorem shows with α_i >2 a all pole TF can be constructed whose magnitude is monotonically decreasing. By adjusting the parameter α_i the desired damping can be achieved.
The generalized time constant τ can be chosen independently of α_i. The coefficients of Butterworth polynomial for an arbitrary τ and a_0 are given below as

$$(14)\ a_1 = \tau a_0$$

(15) $a_i = \dfrac{\tau^i a_0}{\alpha_{i-1}\alpha^2_{i-2}\alpha^3_{i-3}...\alpha^{i-1}_1}$ where $i = 2, 3,..n$

Thus it can be concluded that the coefficients of Butterworth polynomials are dependant on two parameters i.e. characteristic coefficients (α_i) and generalized time constant (τ).

8. DESIGN OF CONTROLLER FOR AN ARBITRARY PLANT

In the feed back configuration the controller can be implemented for an arbitrary plant by using feed back along with a feed forward controller outside the feedback loop. The controller set up is shown in fig. 8 below.

In the block diagram shown below *G(s)* is the plant TF expressed as below.

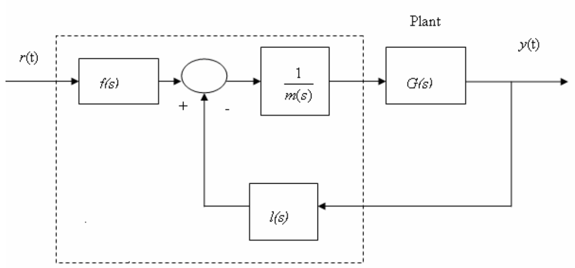

Fig. 8. Block Diagram of Two Parameter Controller

$G(s) = \dfrac{n(s)}{d(s)}$

The close loop transfer function

$T(s) = \dfrac{Y(s)}{R(s)} = \dfrac{f(s)n(s)}{p(s)}$

where $p(s) = m(s)d(s) + l(s)n(s)$

Example-2.5

G(s) is taken here as an arbitrary higher order plant to illustrate the designing of the above controller.

(16) $G(s) = \dfrac{600(s^2+s+2.5)}{s^6 + 4.8s^5 + 12.84s^4 + 11.232s^3 + 65.02s^2 + 213.27s + 1189.3}$

$p(s)$, $m(s)$ and $n(s)$ are expressed as a factor of numerator of *G(s)* are given below as

(17) $$\begin{cases} p(s) = \overline{p}(s)(s^2+s+2.5) \\ m(s) = \overline{m}(s)(s^2+s+2.5) \\ n(s) = \overline{n}(s)(s^2+s+2.5) \end{cases}$$

(18) $$p(s) = \overline{p}(s)(s^2+s+2.5) = \overline{m}(s)(s^2+s+2.5)d(s) + l(s)\overline{n}(s)(s^2+s+2.5)$$

The above equation is known as Diophantine equation (Franklin, *et al.*, 2002) .With a given polynomial *p(s)* the values of the coefficients of numerator and denominator of controller is found out. If the order of *d(s)* is *p*(given) and order of *m(s)* is *q*(to be calculated) then the direct count yields *2q+1* unknowns in *m(s)*, *l(s)* and *(p+q)* equations for the coefficient of power *s*. Then the requirement arises

(19) $2q+1 \geq p+q$ or $q \geq p-1$

From the equation (19) the order of the controller is found out to be $q=5$ .So the 5th order controller polynomials can be expressed as

(20)
$$\begin{cases} m(s) = m_5s^5 + m_4s^4 + m_3s^3 + m_2s^2 + m_1s + m_0 \\ l(s) = l_5s^5 + l_4s^4 + l_3s^3 + l_2s^2 + l_1s + l_0 \end{cases}$$

First the values of α_i are calculated with α_1 =2 for *i=2.3…9* using Theorem-2.The coefficients of $\overline{p}(s)$ is calculated with $a_0 = 1$ and $\tau = 1$ using equation 14 and 15. From equation 17, *p(s)* is calculated. The step response *y(t)* with initial value of $\alpha = 2$ is plotted in the fig. 9 below with overshoot 17.12% .The over shoot is again reduced to 2.3% with increasing value of $\alpha = 2.1$ .Finally it is observed that with further increase the value of $\alpha = 2.25$ offers no overshoot with settling time (where output is 0.99) 1.92 seconds (t_1). If the desired settling time be 1 second(t_2) results $\tau = t_2/t_1$ = 1/1.912 = 0.5208.The results are plotted in fig. 10 for different values of α for a comparative analysis and their behavior in time domain .Then the controller polynomials *l(s)* and *m(s)* as shown in equation 20 are obtained with final value of $\alpha = 2.25$.

From Theorem 2 with $\alpha_1 = 2.25$ *p(s)*is found as

(21) $$p(s) = 0.00008s^{11} + 0.00294s^{10} + 0.05043s^9 + 0.5671s^8 + 4.6085s^7 + 28.s^6 + 130.754s^5 + 455.1694\,s^4 + 1144.7638s^3 + 1950.00s^2 + 2100.s^1 + 1500.00$$

Let us express *p(s)* as follows

(22) $$p(s) = p_{11}s^{11} + p_{10}s^{10} ... + p_1s^1 + p_0$$

Equation 22 can be written as

(23) $$p_{11}s^{11} + p_{10}s^{10} ... + p_1s^1 + p_0 = m(s)d(s) + l(s)n(s)$$

The coefficients of the polynomials *m(s)* and *l(s)* are found out from equation 23 with $\alpha = 2.25$ are given below

(24) $$\begin{cases} m(s) = 0.0000052s^5 + 0.00083s^4 + 0.0590s^3 + 2.3932s^2 + 2.4784s + 5.8324 \\ l(s) = 0.0974s^5 + 2.0274s^4 + 24.237s^3 + 180.569s^2 + 780.305s + 1495.375 \\ f(s) = 1500 \end{cases}$$

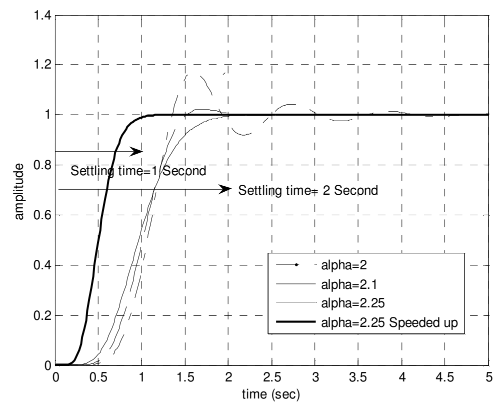

Fig.9. Step Response for different values of α

9. DESIGN OF CONTROLLER FOR PITCH CONTROL SYSTEM OF AN AIRCRAFT

The controller discussed above is redesigned for pitch control system of a BRAVO fighter aircraft (McLean, 1990). As shown in fig. 10 θ_{ref} is the reference pitch angle command as desired by the pilot, δ_E is the elevator deflection angle and θ is the actual output pitch angle.

$$(25)\quad G(s)=\frac{\theta(s)}{\delta_E(s)}=\frac{-20.67(s+0.621)}{s(s^2+1.822\,s+28.54)}$$

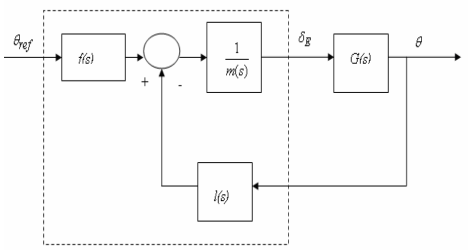

Fig.10. The Pitch Control System of an Aircraft

As per the convention of NASA (Olivera, 2008). the down ward motion of the elevator is known as positive elevator deflection so that positive elevator deflection results positive pitch angle .For our simulation purpose here the negative sign is not considered because we have considered the pitch angle obtained by the negative deflection of the elevator. The order of the plant *G(s)*, i.e. $p=3$ .From equation 19 the order of the controller is found out to be, $q=2$ .So the 2nd order controller polynomials can be expressed as

$$(26)\quad \begin{cases} m(s)=m_2s^2+m_1s+m_0 \\ l(s)=l_2s^2+l_1s+l_0 \end{cases}$$

So the order of the polynomial *p(s)* is , $p+q=5$.As *p(s)* is of order 5, the order of $\overline{p}(s)$ will be of 4 from equation 19.The values of α_i s are calculated with $\alpha_1=2$ for $i=2.3,4$ using Theorem 2.Then the coefficients of $\overline{p}(s)$ is calculated as before with $a_0=1$ and $\tau=1$. Two different values of the α taken here for simulation are 2.1, and 2.25. The first value of α =2.1 produces overshoot of 10.8 %.The overshoot is undesirable for the aircraft which may cause problem for the pilot to maneuver and control it. So the second value of α =2.5 is taken to reduce the overshoot. Now it is observed at α =2.5 overshoot is found to be zero with settling time 2.25 seconds (t_1). As the aircraft possesses faster dynamics the lesser settling time is always preferred. The settling time desired here is 1.25 second (t_2) which results $\tau = t_2/t_1$ = 1.25/2.25= 0.5. The step response for the two different values of α, are plotted along with the fastened response in fig. 11 shown below.

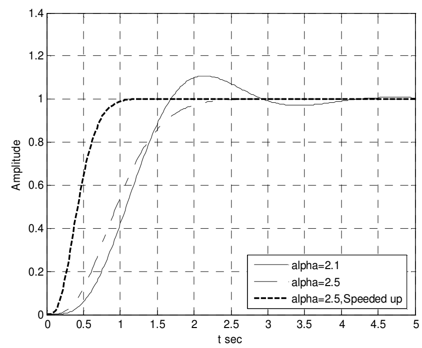

Fig.11. Step Response of the Pitch Control System

The Butterworth polynomial $\overline{p}(s)$ is calculated with $\alpha=2.5$ as done in previous section given below.

$$\overline{p}(s)=0.0002s^4+0.0067s^3+0.0797s^2+0.4464\,s\,+1.0$$

$p(s)$ is calculated multiplying $(20.67s+12.84)$ with $\overline{p}(s)$ given below

$$(27)\quad p(s)=0.004s^5+0.1407s^4+1.7334s^3+1.0251s^2+26.402s+12.840$$

The controller polynomials obtained using equation 23 are given as

$m(s)=0.005\ s^2+1.6933\ s+1.0322$

$l(s)=0.795\ s^2+3.436\ s+12.84$

$f(s)=12.84$

10. KHARITONOV'S STABILITY CRITERIA

In practice the parameters of the aircraft are subjected to certain changes in their original value due to wind speed and change in the aircraft speed . The Kharitonov stability test is carried out here to establish that the system with designed controller is stable and robust to with stand the parametric changes.

Let the characteristic equation *p(s)* which is a monic polynomial (the highest coefficient of *s* =1) is defined as below

(28) $f(s) = a_0 + a_1 + a_2 + ... + a_{n-1}s^{n-1} + a_n s^n$

$\underline{a}_k$ and $\overline{a}_k$ are the smallest and largest value of coefficient of s^k respectively with perturbation $\mu(x)$ in the characteristic equation defined as below

$$\underline{a}_k = a_k - \mu(x)$$

$$\overline{a}_k = a_k + \mu(x) \quad for\ k = 0,1,2,3,..n-1$$

The four monic polynomials obtained as follows:

(29)

$$\begin{cases} \overline{a}_n = \underline{a}_n = 1 \\ g_1(s) = \underline{a}_0 + \overline{a}_2 s^2 + \underline{a}_4 s^4 + ... = \sum_{k=0,even}^{n} (j^k \times \min(j^k \underline{a}_k, j^k \overline{a}_k) s^k \\ g_2(s) = \overline{a}_{0+} \underline{a}_2 s^2 + \overline{a}_4 s^4 + ... = \sum_{k=0,even}^{n} (j^k \times \min(j^k \underline{a}_k, j^k \overline{a}_k) s^k \\ h_1(s) = \underline{a}_1 + \overline{a}_3 s^3 + \underline{a}_5 s^5 + ... = \sum_{k=1,odd}^{n} (j^{k-1} \times \min(j^{k-1} \underline{a}_k, j^{k-1} \overline{a}_k) s^k \\ h_2(s) = \overline{a}_{1+} \underline{a}_3 s^2 + \overline{a}_5 s^5 + ... = \sum_{k=1,odd}^{n} (j^k \times \min(j^k \underline{a}_k, j^k \overline{a}_k) s^k \end{cases}$$

The Kharitonov's polynomials are defined as

(30) $k_{kl}(s) := g_k(s) + h_l(s)$ *, where k,l=1,2*

If the above four polynomials are Hurwitz then the characteristic polynomial is Hurwitz and stable within the given perturbation range. It is assumed here the nominal value of μ to be 20 % of the coefficients of the characteristic equation .The four polynomials with above value of μ are found out from equation 29 and 30 as follows.

(31)

$$k_{11}(s) = 2233.043 + 4591.652s + 2674.252s^2 + 452.217s^3 + 24.486s^4 + s^5$$

$$k_{12}(s) = 2233.043 + 6887.471s + 2674.252s^2 + 301.478s^3 + 24.486s^4 + s^5$$

$$k_{22}(s) = 3349.565 + 6887.478s + 1782.834s^2 + 301.478s^3 + 36.730s^4 + s^5$$

$$k_{21}(s) = 3349.565 + 4591.652s + 1782.834s^2 + 452.217s^3 + 36.730s^4 + s^5$$

$k_{11}(s), k_{12}(s), k_{21}(s)$ and $k_{22}(s)$ in equation 31 are tested for and found to be Hurwitz using Routh's array. So the controller designed for the pitch control system is robust within the perturbation range.

11. CONCLUSION

In the last decade many elegant modern control techniques such as H_∞, H_2 and μ have been developed for designing LTI system. Despite their theoretical success these techniques have drawbacks resulting higher order fragile robust controller in practice. The CRA techniques discussed in this paper establishes a relation between generalized time constant and characteristic ratios relating to the unit step response to select a characteristic polynomial for a desired response. The problems of overshoot reduction and changing the speed of the response are addressed in this paper for arbitrary and real aircraft pitch control system. The use of a Butterworth filter for dynamic systems in terms of the characteristic ratios guarantees stability. Further the problem of parametric perturbation is addressed to make the pitch control system robust using Kharitonov's Criteria.

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