# ON ESTIMATION OF THE ORIENTATION OF MOBILE ROBOTS USING TURNING FUNCTIONS AND SONAR INFORMATION 

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#### Abstract

SONAR systems are widely used by some artificial objects, e.g. robots, and by animals, e.g. bats, for navigation and pattern recognition. The objective of this paper is to present a solution on the estimation of the orientation in the environment of mobile robots, in the context of navigation, using the turning function approach. The results are shown to be accurate and can be used further in the design of navigation strategies of mobile robots.


Keywords: SONAR, Navigation, Pattern Recognition, Signal Processing, and Turning Functions.

## 1. INTRODUCTION

The objective of the work is to estimate the orientation of mobile robots in navigation environments, based on information provided by a SONAR system. The robot is an ATRV-Jr mobile robot, (iROBOT 2002). The work is part of EU project CIRCE - Chiroptera Inspired Robotic Cephaloid: A Novel Tool for Experiments in Synthetic Biology, (CIRCE 2002).

In navigation of a mobile robot the state of the robot has two parameters: its position (which can be coded by ordinary number or by a pair of scalars) coordinates and the orientation of the robot. Estimation of the orientation can be solved in a wide variety of methods with roots in the inverse problems, state estimation or pattern recognition domain. In this work methods from pattern recognition are considered.

The problem of estimation of the orientation will be solved in two steps: first, the estimation of the position, and, second, the estimation of the orientation with reference to the estimated position.

[^0]In section 2 the attributes of the experiments are presented and briefly explained. In section 3 computational details of turning functions and the algorithm for classification are presented. Next, in section 4 some simulation results and, the conclusions are presented.

## 2. DESCRIPTION OF THE EXPERIMENT

The exploratory and working field in this experiment is a corner from our laboratory, where 35 positions were considered. Positions from 1 to 21 are considered reference positions, and the rest, positions from 22 till 35 , are test positions. Two sets of data are obtained: reference data and test data. The distribution of the positions is presented in Fig.1.

The ATRV-Jr mobile robot has a SONAR system with 17 transducers. In every reference position, the robot is making a complete rotation around his center, stopping 12 times for measurements. In every position measurement data from each SONAR sensor are collected and stored for later usage. The structure of reference data, in each position, is a $17 \times 12$ matrix with real values. All the measurements in the reference positions are referenced to the horizontal
axis (by adding the initial orientation angles to the values of angles in each reference position). The data structure of test positions is the same.


Fig. 1. The environment and the distribution of the reference positions (1:21) and test positions (22:35)

Every test array is a pattern matrix of size $17 \times 12$, but the initial orientation of the robot is unknown, and should be estimated.

In Fig. 2 some measurements from a reference position are presented. On the upper side the combined measurements from all orientations and, on the bottom, the measurements from one orientation only, corresponding to an orientation value of zero degrees of the robot.

The considered algorithm compares the SONAR pattern from the test orientation with all the orientation from the closest reference position, to decide which one is resulting in the best match. The usage of an classifier based on Euclidian distance failed in generation of valid results for all the test positions and orientations.

## 3. THE TURNING FUNCTION

The turning function is widely used in the pattern recognition applications, like recognition of polygons


Fig. 2. SONAR data of position \#2: all orientations and the orientation 0 only
(Arkin et al 1991; Alt et al 1995; Alt et al 1996) or indexing of shape images (Chiueh 2000).

The turning function, or cumulative angle function, of a polygon represents the angles of the polygon's edges with respect to a reference axis, of arbitrary orientation as shown in Fig. 3, for a simple polygon. As the perimeter of a polygon is traversed, the angle at each point is computed, thus effectively transforming a 2D shape into a 1D turning function while preserving all relevant information. The starting point on the polygon's contour for the traversal corresponds to the position on the turning function diagram and is called the origin. The origin can be arbitrarily chosen. The above definition is general enough to apply to any type of shape. For closed shapes, the function repeats itself shifted up by $2 * \mathrm{pi}$ in each successive period, which is equal to the length of the boundary.

To support scale invariance, the length of one cycle of the traversal around a polygon is normalized to 1 in the turning function. Note that rotating the reference axis by $\theta$ degrees shifts the entire turning function up or down by $\theta$, depending on the direction of rotation. The same effect occurs if the polygon is
rotated around a point on the plane. Sliding the polygon's origin along the perimeter effectively
shifts the turning function left or right by the amount.


The turning function


Fig. 3. An example of the correspondence polygon - turning function

These two parameters, namely the choice of reference axis and the traversal starting point, can be tuned to accomplish a "maximal" match between two polygons' turning functions. Their distances are then computed at the maximally matched configuration.

For polygons the turning function is a piecewise constant function, increasing or decreasing at the vertices, and constant between two consecutive vertices.

The turning function is invariant to some transformations like translation, rotation and scaling. So it seems to be the ideal candidate in defining a good similarity measure between two polygons.

To compare two polygons whose turning functions are $T_{\mathrm{A}}(s)$ and $T_{\mathrm{B}}(s), s$ in $[0,1]$, a function of the area between the functions should be computed. Then, it is minimized over all choices of reference axis (angle $\theta$ ) and origin for the second polygon ( $t$ ), while keeping the first one's reference axis and origin fixed. First, a function $h($.$) is computed$

$$
\text { (1) } h(\theta, t)=\int_{0}^{1}\left(T_{A}(s+t)-T_{B}(s)+\theta\right)^{2} d s
$$

The distance metric $D\left(T_{A}, T_{B}\right)$ between two turning functions $T_{A}($.$) and T_{B}($.$) is (Veltkamp 2001):$
(2) $D\left(T_{A}, T_{B}\right)=\min \sqrt{h(\theta, t)}$

Modifying the orientation of the reference axis corresponds to rotating the polygon or, equivalently, shifting the turning function up or down.

If the information of the polygon is available under polar representation, which means the angles
(radians) and the range to vertices (meters) of the polygon, a polygon with $n$ vertices is represented by

$$
\begin{aligned}
& \text { (3) } \mathbf{R}=\left[\begin{array}{llll}
R_{1} & R_{2} & \ldots & R_{n}
\end{array}\right] \\
& \text { (4) } \boldsymbol{\theta}=\left[\begin{array}{llll}
\theta_{1} & \theta_{2} & \ldots & \theta_{n}
\end{array}\right]
\end{aligned}
$$

Computation of the turning function can be done in two ways: geometric and algebraic.

## A geometric solution

In Fig. 4 a simple geometric scheme is presented for a polygon with two vertices, $A_{1}$ and $A_{2}$. The origin axis is $O x$. The angle $\theta$ is the difference of angles $\theta_{1}$ and $\theta_{2}$, corresponding of the ranges $R_{1}$ and $R_{2}$.

The problem is to evaluate the $\alpha$ angle, which correspond to a value of the turning function, and the length of the first edge of the polygon, edge defined by the points $A_{1}$ and $A_{2}$. The following relations are valid. For the edge $A_{1} A_{2}$

$$
\begin{gathered}
\text { (5) } \theta=\theta_{1}-\theta_{2} \\
\text { (6) } A_{1} A_{2}=\sqrt{Q A_{1}^{2}+Q A_{2}^{2}} \\
\text { (7) } Q A_{1}=O A_{1}^{\prime}-O A_{2}^{\prime}=R_{1} \cos \theta_{1}-R_{2} \cos \theta_{2} \\
\text { (8) } Q A_{2}=A_{2} A_{2}^{\prime}-Q A_{2}^{\prime}=R_{2} \sin \theta_{2}-R_{1} \sin \theta_{1}
\end{gathered}
$$

$$
\text { (9) } A_{1} A_{2}==\sqrt{R_{1}^{2}+R_{2}^{2}-2 R_{1} R_{2} \cos \theta}
$$

and for the angle $\alpha$

$$
(10)^{\bar{\alpha}}=180-\alpha
$$

(11) $\operatorname{tg} \bar{\alpha}=\frac{Q A_{2}}{Q A_{1}}=\frac{R_{2} \sin \theta_{2}-R_{1} \sin \theta_{1}}{R_{1} \cos \theta_{1}-R_{2} \cos \theta_{2}}$


Fig. 4. The computation of turning function in polar coordinates

$$
\text { (12) } \alpha=180-\operatorname{arctg}\left(\frac{R_{2} \sin \theta_{2}-R_{1} \sin \theta_{1}}{R_{1} \cos \theta_{1}-R_{2} \cos \theta_{2}}\right)
$$

The information necessary in the turning function is related to the angle $\alpha$ and to the length of the edge $A_{1} A_{2}$.

## An algebraic solution

With reference to the bottom of Fig. 4 let be the $\left(x_{i}, y_{i}\right)$ the Cartesian coordinates of the vertices of the considered polygon, with $i$ going from 1 to $n$. and a small drawing like in the Figure 17.b was presented. The following relations hold
(13) $A_{1} A_{2}==\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$

$$
\text { (14) } \alpha=\operatorname{arctg}\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)
$$

As a final remark, the co-domain of the turning function in on $[0: 2 \pi)$, so it is possible to make some offset corrections after directly applying the expressions for the computation of the turning angles.

For the purpose of the present work, the SONAR data from the test position are not ordered, so, for each orientation a turning function is computed. Every position, reference and test, has a set with 12
turning functions corresponding to the 12 orientations of the robot in that position. By taking the index of the minimum distance between the reference and the test turning function the value of


Fig. 5. Results of the estimation of the initial orientation

## 4. SIMULATION RESULTS

In Fig. 5 a global result for the estimation of the initial angle is presented. On the left side of Fig. 5 for every position a pair of two rectangles are represented. On the left is the real value and on the right the estimated value. On the right side of Fig. 5 the absolute errors are represented.

Taking into account the fact that the rotating increment is about 30 degrees ( $360 / 12$ positions) the results are acceptable because the absolute error is less than the 30 degree for each considered position.

There is only one exception, position 29 , which can be explained by observing that a "circle" covers the location and the rotation information cannot be coded in the sonar data and, hence, cannot be extracted by the turning function.

## 5. CONCLUSIONS

The objective of the work was to present a solution to the estimation of the orientation of mobile robots, using SONAR information and techniques from pattern recognition. The turning function is used as it proves to be sensitive in the discrimination of the orientation. Depending on the application, the estimation can be obtained immediately from the SONAR information gotten from a single orientation or can be obtained by averaging all estimates from different orientations of the robot, requiring the robot to rotate on the spot. The results are accurate for almost cases ( 13 from 14 test cases) and less than the step of moving from one orientation to the next one.

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the initial orientation is estimated. To increase the robustness of the estimator, the final orientation of the robot is computed as the mean of all the estimations from the 12 considered orientations.


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